International Journal of Science, Technology, Engineering and Mathematics
Volume 2 Issue 1 March 2022
DOI: https://doi.org/10.53378/352872

Exploring Students' Procedural Fluency and Written Adaptive Reasoning Skills In Solving Open-Ended Problems

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Abstract

Developing students' mathematical skills requires both procedure and reasoning. However, the declination of possessing these skills is still evident today. Hence, this study aimed to describe the students' procedural fluency in terms of accuracy, flexibility, and efficiency and written adaptive reasoning in terms of explanation and justification in solving open-ended problems. The study employed descriptive-correlational design through purposive sampling of thirty students from a National High School in Laguna, Philippines. The quantitative data revealed that in procedural fluency, students can quickly submit a complete solution leading to correct answer. However, they fail to provide two or more solutions in solving open-ended problems. The results also showed that students can clearly explain the problem but struggle to justify their solution. Moreover, procedural fluency is positively correlated to their adaptive reasoning. Consequently, students with an average level of mathematical achievement scored significantly higher than those at a low mathematical level in terms of flexibility. Pedagogical implications suggest that problem-solving activities for students should not solely focus on getting the correct procedures and answers. Further, it is recommended that teachers should expose students in open-ended problems and allow them to try and justify their own unique solutions irrespective of their mathematical achievement.

Keywords: *mathematical achievement, open-ended problems, procedural fluency, problem-solving, written adaptive reasoning*

Received: January 8, 2022 **Reviewed:** January 29, 2022 **Accepted:** February 21, 2022

Suggested Citation: Andal, S.B. & Andrale, R.R. (2022). Exploring Students' Procedural Fluency and Written Adaptive Reasoning Skills in Solving Open-Ended Problems. *International Journal of Science, Technology, Engineering and Mathematics*, Volume 2 Issue 1, pp. 1 - 25. DOI: https://doi.org/10.53378/352872

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1. Introduction

Education is often considered a real success behind any future success. It paves the way for the people to have a promising future and to receive ample opportunities along the way (Al-Shuaibi, 2014). Therefore, Philippine education implemented its educational reform called the K to 12 program to achieve its unending pursuit of a better educational system and ensure that every Filipino student receives a high-quality basic education. One of the major fields of the current curriculum for basic education is Mathematics (Department of Education, 2016). It is given priority for being a queen of all sciences.

Even after the implementation of K to 12 program, the Philippines' first participation in Programme for International Students Assessment (PISA) in 2018 resulted to Filipino students achieving lower than the average points expected from the Organisation for Economic Co-operation and Development (OECD). It was shown that among 79 countries, the Philippines ranks 70s in mathematics and science. In addition, the National Achievement Test (NAT) results in 2017-2018 also showed that mathematics has the lowest mean percentage score, and problem-solving is still way below the acceptable mean percentage score. Therefore, the need to improve the problem-solving skills in Mathematics is not just important but an urgent matter that every student and educator must address.

Problem-solving is said to be the heart and soul of mathematics (Stupel & Ben-Chaim, 2017). The National Council of Teachers of Mathematics (NCTM) recommends that problem-solving be the focus of mathematics teaching since its significance in everyday living cannot be denied. It was also emphasized that problem-solving is the application of knowledge, which must be taught in various ways and strategies. Bernard and Chotimah (2018) acknowledged that one of the avenues that would extend the students' problem-solving skills is allowing them to answer an open-ended problem with various answers and solutions. Open-ended problems are questions with multiple solutions and answers (Milos, 2014). These are rarely utilized in mathematics classrooms since most teachers rely on textbooks for instruction, which use only closed-ended questions with examples and exercises that all have the same solution and fixed answer.

According to Noureen et al. (2015), one way to assess students' problem-solving abilities is to investigate their level of mathematical proficiency. Mathematical proficiency is composed of five interconnected strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Accordingly, Bernard and Chotimah (2018) also emphasized that using an open-ended approach may instill students' mathematical reasoning ability and promises an opportunity for students to use the various strategies, procedures, and ways that they believe will fit in a given question. It implies that exposure to open-ended questions would enhance students' procedural fluency and adaptive reasoning, which are parts of the five interconnected strands in mathematical proficiency. In addition, Rizki et al. (2017) also highlighted both procedural fluency and adaptive reasoning as requirements in developing students' mathematical ability. Procedural fluency is said to be the foundation among all the strands of mathematical proficiency (NCTM, 2014). Inayah et al. (2020) relates procedural fluency to students' comprehension of mathematical ideas and problems which Foster (2017) refers to as fundamental in students' mathematical development. On the other hand, adaptive reasoning involves students' ability to think logically about relationships between mathematical concepts (Dewi et al., 2020). As explained by Muin et al. (2018), it is a vital skill in learning mathematics that demonstrates learning ability.

In today's academic landscape, students find it challenging to use an accurate and flexible method in solving a problem. They also lack the ability to reason out and even expound their solutions. It was particularly confirmed by the study of Aprianti (2014) that no students had provided a fluent mathematical procedure. Similarly, Asmida (2016) revealed that the adaptive reasoning of the students was in the middle average only. Numerous researchers have also demonstrated a significant association between students' procedural fluency and adaptive reasoning (Bautista, 2012; Dewi et al., 2020).

This study aims to determine the relationship between the level of procedural fluency and adaptive reasoning skills of the students. Parallel to the studies of Dewi et al. (2020) and Bautista (2012) which used geometric proofs and thermodynamic problems in physics, this study explores application of the skills on Grade 10 mathematics. Supported by the study of Awofala (2017) that students' mathematical achievement is significantly related to all strands of mathematical proficiency, this study also fills the literature gap by exposing students to open-ended questions in mathematics, which provide opportunities to learn diverse solutions and strategies regardless of their mathematical achievement. Specifically, it aims to determine the students' level of procedural fluency in solving open-ended problems in terms of accuracy, flexibility and efficiency and the level of written adaptive reasoning skills in solving open-ended problems as to explanation and justification. It will also provide answers to the following hypotheses:

Ho1: Students' procedural fluency significantly relate to their written adaptive reasoning skills in solving open-ended problems.

Ho2: Students' procedural fluency significantly differ to their written adaptive reasoning skills when grouped according to mathematical achievement.

2. Literature review

2.1. Procedural Fluency

Among the five strands of mathematical proficiency, procedural fluency is said to be the foundation of all strands (NCTM, 2014). Procedural fluency, sometimes called "smooth procedural" and "mathematical fluency" involves knowledge on when and how to apply a method, strategy, or procedures and being able to accurately, efficiently, and flexibly. Foster (2017) states that achieving procedural fluency is fundamental in students' mathematical development. Moreover, repetitive exercise is a common technique to develop this ability. Laswadi et al. (2016) mentions that learning experiences greatly help students to construct procedures.

In order to develop procedural fluency, students need experience in integrating concepts, processes and building on familiar methods as they create their own informal strategies and procedures (NCTM, 2014). Students need opportunities to justify both informal strategies and commonly used techniques mathematically, support and justify their choices of appropriate processes, and strengthen their understanding and skill through distributed practice. Thus, augmenting students' procedural fluency in mathematics is gravely important.

In the study of Bautista (2012), students' procedural fluency is influenced by their mathematical ability, whereas their written-mathematical explanation varies depending on their English ability. In addition, Dewi et al. (2020) analyzed the adaptive reasoning abilities and procedural fluency of students in a study which showed that students can think logically in choosing the right concepts and situations. Inayah et al. (2020) also found that students can solve problems with more than one method but some students are incapable to do streamline steps and make accurate calculations. This is in parallel to the findings of Aprianti (2014) and Asmida (2016) that none of the students had the smooth mathematical procedure and procedural fluency which impede their mathematical development. All these studies used accuracy, flexibility, and efficiency as parameters in measuring the level of procedural fluency of the students.

*Accuracy***.** It is the ability of the students to obtain the correct answer without committing any mistakes. However, several researchers agreed that accuracy is the most difficult indicator of procedural fluency for students to achieve (Inayah et al., 2020; Aprianti, 2014; Asmida 2016). The results of their studies showed that accuracy was the lowest achievement compared to the other indicators. Most of the students did not meet the expected accuracy in problem-solving. They made continuous mistakes because they lack interest in re-checking their solutions and answers.

*Flexibility***.** According to Inayah et al. (2020), flexibility is the students' ability to recognize strategies necessary to complete a mathematical task and to apply learned strategies to alternative mathematical tasks - particularly in solving problems. As applied in Inayah et al. (2020) study, students' flexibility is taken as students' capability to determine strategies in problem-solving and carry out problem-solving procedures using known methods. The study shown that this indicator was the highest achievement indicator compared to other indicators. Most of the students can provide more than one way, strategy, or solution to solve specific problems.

*Efficiency***.** As defined by Asmida (2016), efficiency is the ability of the students to provide a strategy in the quickest way to solve problems. Inayah et al. (2020) found in a study that some students were unable to immediately provide correct procedures or solutions in a shortest possible time. In addition, there were some students who yielded the correct answer but employed an incorrect strategy. This suggested that this indicator was on the average achievement of the students.

2.2. Written Adaptive Reasoning

Developing students' mathematical skills requires both procedures and also reasoning (Rizki et al., 2017). According to Muin et al. (2018), reasoning is a major component in mathematics and should be emphasized as a foundation of mathematics. Adaptive reasoning is also one of the five strands of mathematical proficiency. Dewi et al. (2020) defined adaptive reasoning as students' ability to think logically about relationships between mathematical concepts; a process to justify work (Wibowo, 2016) and a fact, procedure, concept and mathematical solution to be adapted to the situation (Syukriani et al., 2017). According to Muin et al. (2018), it is one of the mathematical skills that must be possessed by students to demonstrate their learning ability. Its importance has been recognized by several researchers.

Muin et al. (2018) studied and used intuitive-inductive (guess and make general conclusion) and intuitive-deductive (guess and make logical conclusion) as indicators of adaptive reasoning. The study showed that these two indicators have only slight differences and almost balanced. It can be said that the creative problem-solving learning model is effective learning to develop students' mathematical adaptive reasoning skills. Moreover, Syukriani et al. (2017) also investigated the adaptive reasoning and strategic competence of a male and female student as they solved mathematical problems and found that male and female subjects used different strategies to understand, formulate, and represent a problem situation. The study also used explanation and justification as their standards in measuring the level of adaptive reasoning of the students.

*Explanation***.** Andrews et al. (2019) asserts that the act of explaining can help students develop new understandings of mathematical ideas, construct rules for solving problems, become aware of misunderstandings or a lack of understanding, and develop their mathematical communication. The students' explanations can also offer opportunities for a teacher to understand more fully what the students are thinking.

*Justification***.** Analyzing students' justification skills allows teachers to study the development of mathematical understanding and create a learning design that helps students on how to justify their answers (Eko et al., 2018). The ability of a person to justify is closely related to his reasoning ability because justification means giving reasonably clear reasoning. Furthermore, justification is the process of validating a statement by giving reasons, proven definitions, or theorems. Thomas (2018) adds that justification is the act of providing a foundation, proofs, or arguments to convince another person that a claim is true.

2.3. Mathematical Achievement

Mathematics plays a vital role in most careers. However, DepEd, PISA, and NAT proven that it remains the concern in the Philippines. Carey et al. (2017) affirm that high level of mathematical skills have long been recognized as essential not only for academic success but also for efficient functioning in everyday life. However, Arigbabu (2013) proved that many students performed poorly in both internal and external examinations in mathematics which explains their lack of mathematical proficiency. Among the mathematical skills, procedural fluency and adaptive reasoning also need to be addressed in assessing students' performance. Awofala (2017) found a significant positive correlation between the student mathematical achievement and all components of mathematical proficiency which includes procedural fluency and adaptive reasoning. Students reach a high level of these components when they are on the above average level.

2.4. Open-Ended Problems

In order to develop students' mathematical ability, mathematics learning should provide students with the opportunity to freely try their own possible solutions (Kwon et al., 2016). Dewi et al. (2020) argued that a problem could only boost students' thinking if it cannot be solved via a well-known routine approach. In this perspective, many researchers (Kwon et al., 2016; Levav-Waynberg & Leikin, 2012; Wijaya, 2017; Wessels, 2014) suggested using open-ended problems to stimulate students' divergent thinking and mathematical creativity.

According to Wijaya (2017), open-ended problems provide students with the chance and stimulus to investigate alternative solutions, methods, or tactics. Hadiastuti et al. (2019) added that there are three types of open-ended problems: problems with multiple answers, problems with many ways of solving a problem, and problems that can be developed into new problems. However, Albab and Wangguway (2020) mentioned only two types of open-ended problems: one answer with many ways of solutions, and the other is several solutions with many answers. These kinds of problems are valuable since they allow students to learn new methodologies, enhance their mathematical knowledge, and develop their mathematical creativity (Yuniatri et al., 2017). Furthermore, they stated that open-ended problems are tools that contain references to various types of knowledge, different levels of complexity in mathematical thinking, and multiple levels of creative thinking in its various dimensions (fluency, flexibility, complexity, and creativity).

Wijaya (2017) recommended the use of open-ended problems in honing students' mathematical creativity. It was further affirmed that students' mathematical creativity can be gauged by the students' ability to employ various solutions or strategies in problem-solving. Albab and Wangguway (2020) used open-ended problems and found out that students have quite good creative and innovative thinking skills; they can use several ways to solve problems, they are able to find unique ideas to solve problems, and they are able to expand, select, analyze and evaluate the basic idea of problem-solving. Furthermore, Belecina and Ocampo (2018) found that students' critical thinking improved significantly after using problems, and positive attitudes and feelings were apparent among students after its use.

The idea of using open-ended problems strongly agrees with the fact that it breaks the stereotype that every problem has only one answer and one solution. Though there are lots of positive results in using open-ended problems in the classroom, its application is not predominant in the classroom. Nold (2017) mentioned that teachers lack the necessary knowledge, skills, and tools to promote a higher level of critical thinking and problem-solving which hinder them to evaluate different solutions, methods, strategies, and answers provided by students. Thus, exposure to open-ended problems and training in employing several heuristics in problem-solving are deemed necessary.

3. Methodology

The study utilized descriptive-correlational with a comparative research design. The level of procedural fluency (in terms of accuracy, flexibility, and efficiency) and written adaptive reasoning (in terms of explanation and justification) of the grade 10 students in solving open-ended problems, their relationship and differences when they are grouped according to their level of mathematical achievement were described and analyzed.

Thirty respondents were chosen using purposive sampling. Due to pandemic, it is critical to address students' internet connectivity. Ten students for each level of mathematical achievement were considered as samples. These students were all from Grade 10 students of Masaya Integrated National High School, Bay, Laguna, Philippines. They were categorized into three levels of mathematical achievement, namely: high, average and low.

Table 1

Profile of the Students According to their Mathematical Level

Grades	Mathematical Achievement		Verbal Interpretation
	f	%	
91-100	10	33.3	High
81-90	10	33.3	Average
Below-80	10	33.3	Low
Total	30	100.0	

As shown in table 1, the mathematics grades of the students were equally distributed into three levels of mathematical achievement, namely: high, average, and low. Each level consists of 10 students, or 33.33% of the respondents, and a total of thirty Grade 10 students with 18 females and 12 males.

The details of the purpose and development of the instruments are the following:

Open-ended Problems. The study employed five researcher-made open-ended problems. Each problem includes guide questions in answering to determine the level of students' procedural fluency skills in terms of accuracy, flexibility, and efficiency; and written adaptive reasoning skills as to explanation and justification. These problems consist of two algebraic word problems, two probability problems, and one figure to arrange open-endedly.

Procedural Fluency and Adaptive Reasoning Scoring Rubrics. Two scoring rubrics were used to describe the students' procedural fluency and adaptive reasoning. The study used a researcher-constructed scoring rubric to analyze the students' procedural fluency in terms of accuracy, flexibility, and efficiency in solving open-ended problems. A scoring rubric was also created to analyze the adaptive reasoning skill of the students in terms of their explanation and justification.

Validation. Prior to the conduct of the study, the seven open-ended problems and the scoring rubrics were evaluated by the experts consisting of six teachers from different High Schools in Laguna, Philippines. The experts consist of five Mathematics teachers and one English teacher who focused on the grammar of the instruments. Moreover, the instrument underwent pilot testing to determine its reliability. Based on the test reliability result, it was found out that the problem-solving test obtained an acceptable level of reliability.

Final Revision. All the five open-ended problems got the acceptable mean score to include in the study. The suggested sentence constructions and grammars were strictly followed for the two rubrics prior to the implementation.

Several panels of experts evaluated the study to ensure the quality of the content. Prior to the conduct of the study, suggestions and comments were carefully examined.

An approval letter and request permission from the principal of the school was sought. After receiving consent to perform the study, the master list of students with their math grades was obtained in order to attain the target number of students in each level of mathematical achievement. Due to the pandemic, the study was conducted with the supervision of Grade 10 head teacher and mathematics teacher through the use of an internet platform, specifically Facebook messenger. The study's objectives were communicated to the selected respondents. The test problem material, which comprised of five-open ended problems, was distributed to the grade 10 head teacher via their platform. The test paper contains detailed instruction for solving the problem as well as the researcher's contact information for queries and clarifications. The head teacher allotted two weeks for students to complete the test problems. After two weeks, the researcher collected data directly from the students in conformity to their teachers' instruction.

In analyzing the level of procedural fluency and written adaptive reasoning, mean, standard deviation, frequency and percentage were used. To determine whether there is a significant relationship between procedural fluency and adaptive reasoning of the students in solving open-ended problems, Pearson-product – moment correlation was employed. Furthermore, one-way ANOVA and Scheffe Post Hoc Test Analysis were applied to identify if significant difference exists between the level of procedural fluency and adaptive reasoning of the students when they are grouped according to their mathematical achievement.

4. Findings and Discussion

Table 2

Students' Overall Procedural Fluency

	Mean	SD	Verbal Interpretation
Procedural Fluency			
Accuracy	16.70	6.43	Moderate
Flexibility	10.03	4.00	Moderate
Efficiency	15.30	6.18	Moderate
Over-all Mean	**14.01**	**5.53**	**Moderate**

Table 2 reveals that over-all students' procedural fluency is at moderate level with a mean score of 14.01 and standard deviation of 5.53. This indicates that there are more students who can quickly submit a complete solution that leads to the correct answer. This finding asserts Inayah et al. (2020) that the level of mathematical procedural fluency of the students has an average performance which is in the moderate category. However, it is apparent that flexibility has the lowest mean score which implies that students struggle in providing two or more solutions.

Table 3

Students' Scores in Procedural Fluency in Solving Open-Ended Problems as to Accuracy

Score	Accuracy		Verbal Interpretation
	f	%	
17-25	16	53.3	High
9-16	10	33.3	Moderate
0-8	4	13.3	Low
Total	30	100.0	

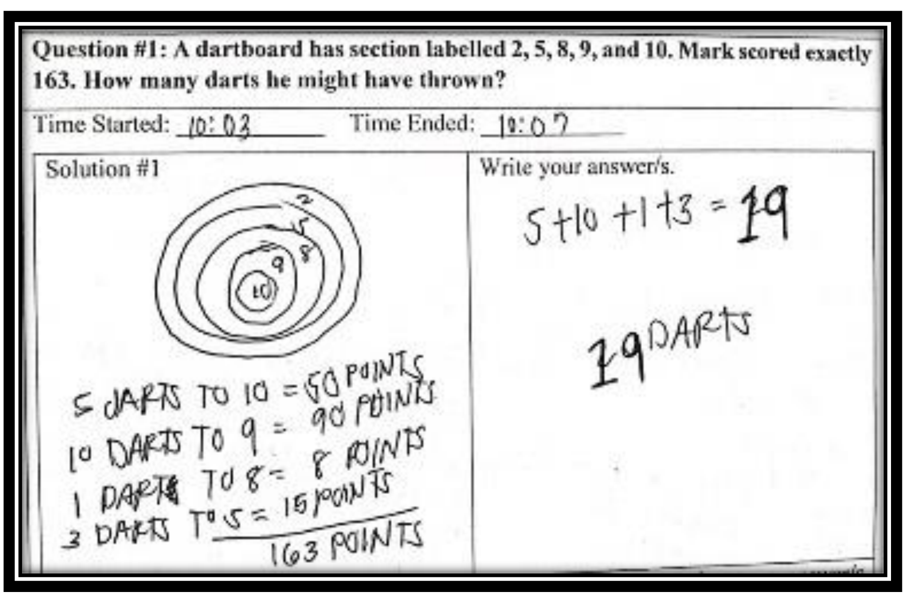
It can be gleaned from table 3 that in accuracy, most of the students in solving open-ended problems got 17-25 points, with a total of 16 students or 53.33% of the respondents. It can also be noted that out of these 16 students, 4 students (Student 9, 15, 16, 23) got perfect scores, and three of them belong to the average level of mathematics achievement. As a result, majority students have a high level of accuracy. This implies that students can provide correct answers with complete solutions in open-ended problems.

This supports the study of Glass and Kang (2020) that students can provide accurate answer in an activity since they tend to look for the accurate answer on the internet or other sources at home. However, the result contradicts the study of Inayah et al. (2020) which shows that accuracy is the most challenging component in procedural fluency. It can be inferred that students commit mistakes since they do not have a chance to re-check their answers as reflected in the guided question section in the test material.

Student 15 is an example of a student with a high level of accuracy. His solution to Problem 1 is depicted in figure 2.

Figure 2

Sample Solution of Student 15 in Problem 1



It is shown that Student 15, who is in the average level of mathematical achievement, was able to solve the problem by providing a complete solution and correct answer. Since the problem asked for the possible number of darts, he multiplied each point of the dartboard to a possible number of darts and added these points to come up with 163 points. It can also be noticed that aside from the solution, he illustrates a dartboard with corresponding points from the problem as he solved it. This figure confirms the study of Al-Balasi and Barham (2010) that students can employ a variety of mathematical representations to enhance their problem-solving mathematical abilities. Pentang et al. (2021) emphasized that discovering multiple representations leads to effective and efficient problem-solving.

Table 4

Students' Scores in Procedural Fluency in Solving Open-Ended Problems as to Flexibility

Score	**Flexibility**		**Verbal Interpretation**
	f	%	
17-25	1	3.3	High
9-16	20	66.7	Moderate
0-8	9	30.0	Low
Total	**30**	**100.0**	

It can be depicted from table 4 that in flexibility, majority of the students in solving open-ended problems got 9-16 points, with a total of 20 students or 66.67% of the respondents interpreted as having a moderate level of flexibility. This summarizes that the students can apply only one strategy leading to the correct procedure and answer. They had difficulty producing more than one solution, limiting them in using one strategy. Akin with Pentang et al. (2021), this can be attributed to the scarcity of students' knowledge on different heuristics and satisfaction once they got the correct answer which led them in providing a single algorithm in problem-solving.

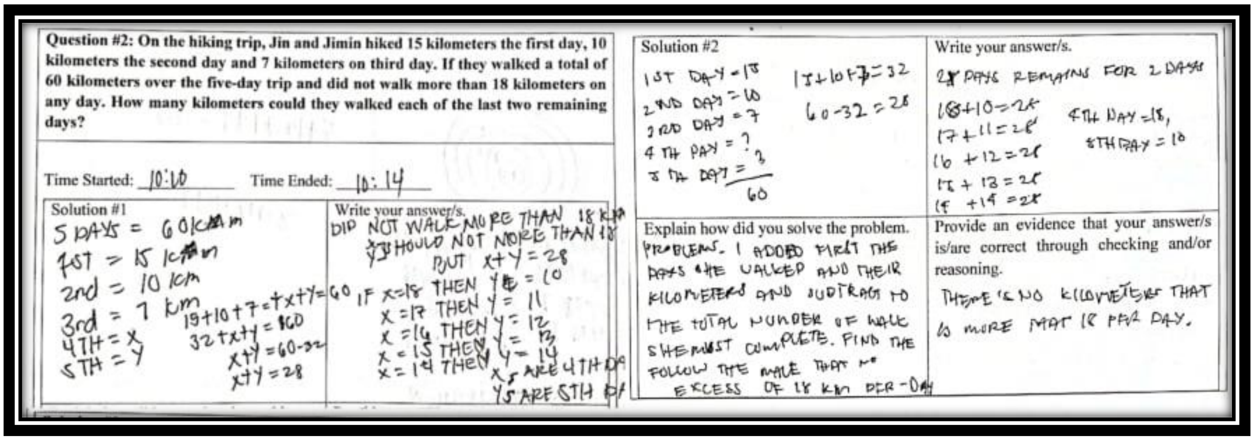
This finding affirms Brookes (2015) who found that students have difficulties providing two or more solutions since they lack motivation and interest to do it once they already arrived at the correct answer on their first solution. He added that flexibility can be fully maximized with the teachers' aid by exposing them to several strategies. In addition, Schukajlow and Krug (2014) concluded that developing students' ability to provide multiple solutions significantly improves mathematical knowledge. This infers that in order for students to develop a high level of flexibility focused on providing multiple solutions in a problem, educators should put emphasis on providing open-ended problems that allow students to apply multiple solutions throughout the instruction.

It can be further noted that one of the thirty respondents got 17-25 points interpreted as a high level of flexibility. This denotes that this student could apply more than one strategy leading to the correct procedure and answer who belong to the average level of mathematical achievement. Therefore, it can be deduced that having a high level of mathematical achievement does not always guarantee a high level of flexibility.

The solutions provided by Student 23 can be seen in figure 3.

Figure 3

Sample Solution of Student 23 in Problem 2



Student 23 belongs to the average level of mathematical achievement who provided more than one solution to this problem. She employed guess and check strategy by applying several computations while staying below the 18-kilometer limit specified by the given problem. Besides, she executed linear equation using x and y as variables. She created an equation such as 15+10+7+x+y=60 which led to x+y=28 that aided her to yield the correct values.

Table 5

Students' Scores in Procedural Fluency in Solving Open-Ended Problems as to Efficiency

Score	Efficiency		Verbal Interpretation
	f	%	
17-25	13	43.3	High
9-16	12	40.0	Moderate
0-8	5	16.7	Low
Total	30	100.0	

It can be gleaned from table 5 that in efficiency, most of the students in solving open-ended problems got 17-25 points, with a total of 13 students or 43.33% of the respondents. As a result, there are more students who have a high level of efficiency. This implies that students can accomplish the task with no errors in the shortest possible time (≤ 5 minutes).

As mentioned by Best (2020), students can immediately answer it because they possess the metacognitive ability to constantly identify the best appropriate techniques for the given challenge. This also demonstrates their flexibility in providing a single answer to a specific problem. In conclusion, having a precise solution in mind for a certain problem enables students to solve problems quicker.

Table 6

Students' Overall Written Adaptive Reasoning

	Mean	SD	Verbal Interpretation
Written Adaptive Reasoning			
Explanation	12.77	6.34	Moderate
Justification	11.63	7.51	Moderate
Over-all Mean	**12.20**	**6.93**	**Moderate**

Table 6 displays that over-all students' written adaptive reasoning is at moderate level with a mean of 12.20 and a standard deviation of 6.93. It means that there are more students capable of explaining and justifying their answer in solving open-ended problems. Similarly, Asmida (2016) found out that the adaptive reasoning of the students was in the middle average only.

Table 7

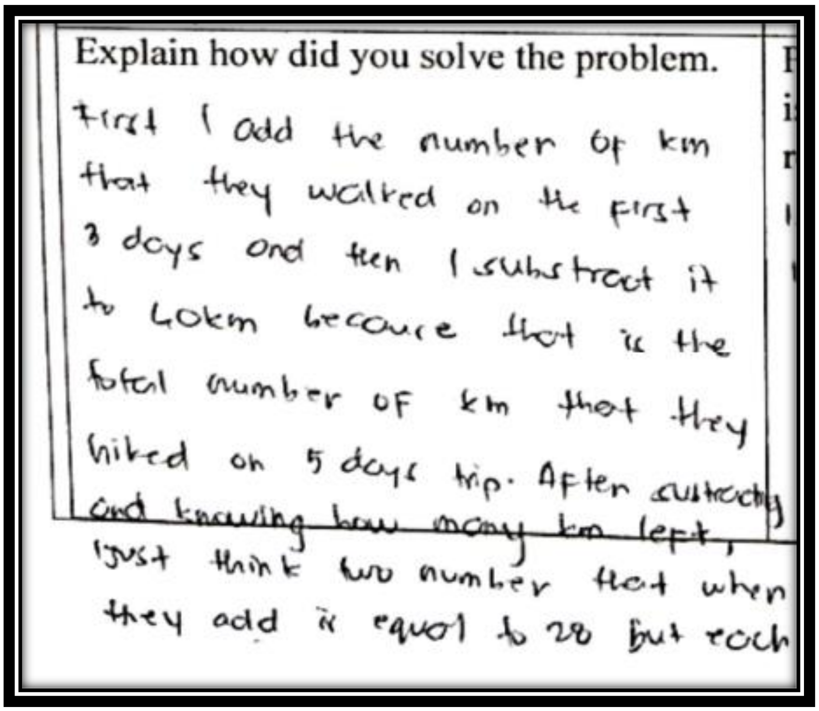
Students' Scores in Written Adaptive Reasoning in Solving Open-Ended Problems as to Explanation

Score	Explanation		Verbal Interpretation
	f	%	
17-25	9	30.0	High
9-16	14	46.7	Moderate
0-8	7	23.3	Low
Total	**30**	**100.0**	

Table 7 demonstrates that in explanation, most of the students in solving open-ended problems got 9-16 points, with a total of 14 students or 46.67% of the respondents. As a result, there are more students who possess a moderate level of explanation. This implies that students are capable of explaining how they arrived at their solution to each problem. However, it can be observed that they continued to make errors in terms of offering a thorough explanation in which they can incorporate other concepts essential to adequately explain their solution to a particular problem. To elaborate on this assertion, figure 4 shows the explanation of Student 26.

Figure 4

Sample Explanation of Student 26 in Problem 2


Explain how did you solve the problem.

First I add the number of km that they walked on the first 3 days and then I substract it to 40km becouce that is the total number of km that they hiked on 5 days trip. After sustracting and knowing how many km left, I just think two number that when they add is equal to 28 but each

As shown in figure 4, the explanation provided by Student 26 was simply in sentence form kind of solution. She started her explanation by the solution she had provided and not by how she came up with that solution. Hence, it indicates that she has provided a clear explanation but not a detailed one.

This finding confirms the study of Bautista (2012) in which the mathematical explanation of the students is still not achieved since cognition in mathematics has a great impact on it. The students lacked cognition about the concepts and ideas in mathematics. Additionally, Andrews (2019) noted that students do not have the ability to provide an explanation in mathematics since they incapable to expand their comprehension of mathematical ideas, unable to construct rules for solving problems, and develop their mathematical communication. This suggests that explanation is critical in the process of mathematics learning. Thus, the teacher’s continuous provision of tasks is deemed necessary to fully develop the students' explanation for their responses.

Table 8

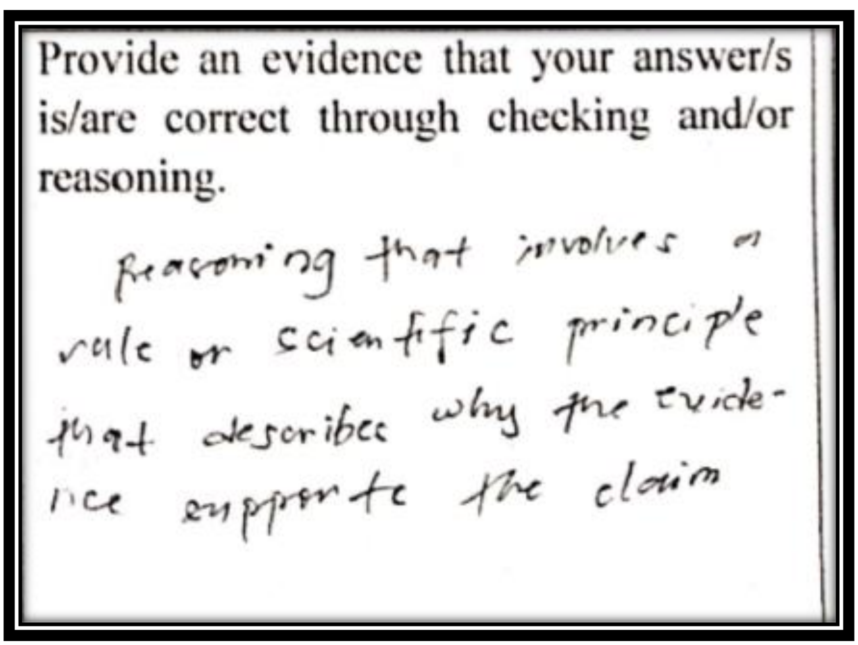
Students' Scores in Written Adaptive Reasoning in Open-Ended Problems as to Justification

Score	Justification		Verbal Interpretation
	f	%	
17-25	10	33.3	High
9-16	8	26.7	Moderate
0-8	12	40.0	Low
Total	**30**	**100.0**	

Based on the table, most of the students in solving open-ended problems got 0-8 points, with 12 students or 40% of the respondents. Consequently, there are more respondents who have a low level of justification. This reveals that students lack the ability to provide clear evidence that supports their answer, whether it is reasoning or checking.

Figure 5

Sample Justification of Student 4 in Problem 3


Provide an evidence that your answer/s is/are correct through checking and/or reasoning.

Reasoning that involves a rule or scientific principle that describes why the evide- nce supports the claim

Evidently, figure 5 shows that Student 4 provides a statement for justification that is totally irrelevant in supporting her answer. Likewise, she demonstrated a piece of unclear evidence that justifies her response to the given problem. Thus, Student 4 is regarded to have a low level of adaptive reasoning in terms of justification.

This is in consonant to the findings of Eko et al. (2018) in which they conducted a study to measure the level of justification ability of the students. From level 3 to 1, they found out that 66% of the students are in level 1, which means students provided unnecessary statements that are irrelevant to the given problem and displayed wrong mathematical concepts to justify their answers. A high level of justification is achieved when students comprehend the proper application of several mathematical concepts in defending their answers.

Table 9

Relationship between the Students' Procedural Fluency and Written Adaptive Reasoning in Solving Open-Ended Problems

Procedural Fluency	Written Adaptive Reasoning	
	Explanation	Justification
Accuracy	.712**	.687**
Flexibility	.497**	.494**
Efficiency	.722**	.702**

*** Correlation is significant at the 0.01 level (2-tailed)*

As shown in table 9, it was found that a high positive significant relationship exists between procedural fluency (in terms of accuracy, flexibility, and efficiency) and adaptive reasoning (in terms of explanation and justification) of the students. This finding indicates that the existence of high or low level in procedural fluency is linked to the occurrence of high or low level in adaptive reasoning. Furthermore, improving the students' adaptive reasoning can be highly associated with enhancing students' procedural fluency. Teachers should give paramount attention in intensifying the students' procedural fluency for it is highly correlated to their adaptive reasoning.

The result affirms the study of Mellony and Stott (2012) that procedural fluency and adaptive reasoning complement each other, and when it happens, a balance of knowledge and the connection between mathematical understanding and computational proficiency is required. This explains that students can provide strategies in solving the problems for they possess enough conceptual understanding. Higher level of mathematical proficiency is only obtained if the students totally acquire the lower level of mathematical proficiency.

Table 10

Difference on the Students' Procedural Fluency and Written Adaptive Reasoning

		Sum of Squares	df	Mean Square	f	Sig.
Procedural fluency						
accuracy	Between Groups	178.400	2	89.200	2.361	0.113
	Within Groups	1019.900	27	37.774		
	Total	1198.300	29			
flexibility	Between Groups	115.467	2	57.733	4.486	0.021
	Within Groups	347.500	27	12.870		
	Total	462.967	29			
efficiency	Between Groups	135.200	2	67.600	1.880	0.172
	Within Groups	971.100	27	35.967		
	Total	1106.300	29			
Adaptive reasoning						
explanation	Between Groups	140.867	2	70.433	1.853	0.176
	Within Groups	1026.500	27	38.019		
	Total	1167.367	29			
justification	Between Groups	198.867	2	99.433	1.867	0.174
	Within Groups	1438.100	27	53.263		
	Total	1636.967	29			

Table 10 reveals no significant difference in procedural fluency in terms of accuracy and efficiency between levels of mathematical achievement. Likewise, there is no significant difference in adaptive reasoning as to explanation and justification between levels of mathematical achievement. This can be inferred that regardless of students' mathematical achievement, it does not affect their performance to provide accurate answers with complete solutions in a shortest possible time. Moreover, their ability to provide an explanation and justification was not influenced by their mathematical achievement.

This is relevant to the study of Brezavšˇcek et al. (2020) that the students' gender and achievement do not affect their performance in mathematics. They found out that attitudes toward mathematics and how they value the importance of the subject greatly impact students' mathematical performance.

The result also shows a significant difference in procedural fluency in terms of flexibility between levels of mathematical achievement with p-value of 0.021. This displays that students' ability to provide two or more strategies in solving open-ended problems varies according to their mathematical achievement level.

Table 11

Post Hoc Test Analysis on Flexibility of the Students in Solving Open-Ended Problems

Dependent Variable			Mean Difference	Std. Error	Sig.	95 % Confidence Interval Lower Bound	Upper Bound
Flexibility	High	Average	-2.60	1.60	0.286	-6.76	1.56
		Low	2.20	1.60	0.403	-1.96	6.36
	Average	High	2.60	1.60	0.286	-1.56	6.76
		Low	4.80	1.60	0.021	0.64	8.96
	Low	High	-2.20	1.60	0.403	-6.36	1.96
		Average	-4.80*	1.60	0.021	-8.96	-0.64

**. The mean difference is significant at the 0.05 level*

In table 10, flexibility is the only component of procedural fluency which showed a significant positive difference when the students were grouped according to their level of mathematical achievement. Table 11 presents the Scheffe Post Hoc Test Analysis on the students' flexibility in solving open-ended problems.

The table displays that in procedural fluency in terms of flexibility, the scores of students' high level of mathematical achievement has no significant difference to average and low level of mathematically achievement students. This implies that students who belong to the high level of mathematical achievement have also experienced difficulty exhibiting at least two strategies in problem-solving. The majority of the students on a high level of mathematical achievement got the same score in the flexibility compared to the low and average level of students in mathematical achievement. It can be concluded that having a high level of mathematical achievement does not guarantee a huge difference in the score in the flexibility as compared to low and average students. This is in contrast to Kattou et al. (2012) that a positive correlation exists between mathematical creativity and ability. The result contradicts that mathematical ability impacts students' creativity in terms of fluency, flexibility and originality.

It is also shown that the average and low scores have a significant difference with a p-value of 0.021. It reveals that those students in the average level of mathematical achievement scored significantly higher than those at a low mathematical level. It can be noted that only one student achieved a high level of flexibility who belongs to an average level of mathematical achievement. As shown in table 3, three out of four students who got perfect scores in accuracy belong to an average level of mathematical achievement. It can be inferred that the same student exhibits both high level of flexibility and a perfect score in accuracy. With further association of the results to the personal profile of the respondent, this particular student is an officer of a mathematics club in their school and an active participant of MTAP sessions every year before the pandemic. This concludes that attending mathematics session can assist students in developing multiple answers to a single problem resulting to much higher score on the flexibility scale. This supports Andrade and Fortes' (2019) results that students' exposure to a variety of training sessions and competitions increases their mathematical creativity which includes flexibility. Furthermore, the claim affirms the findings of Schukajlow and Krug (2014) which found positive results in terms of students' achievement, interest, and motivation, as well as their ability to provide multiple solution. Students who maintain a high-grade point average are more likely to provide multiple solutions. Likewise, Achmetli and Schukajlow (2019) claimed that the students' ability to construct multiple solutions while solving real-world problems is influenced by their achievement and interest before and during the procedures.

5. Conclusion

The main purpose of this study is to describe the level of students' procedural fluency and written adaptive reasoning skills. It was found out that students can provide accurate answers with complete solution in a quickest amount of time. However, they struggle in providing more than one solution in solving open-ended problems. The study also revealed that a highly positive significant relationship exists between students' procedural fluency as to accuracy, flexibility, efficiency, and written adaptive reasoning as to explanation, and justification. Consequently, there is a substantial difference in the students' procedural fluency in terms of flexibility according to mathematical achievement.

Students should be taught general mathematical problem-solving skills, but the precise strategies should be left for them to discover. Similarly, in classroom drills, teachers should allow students to choose any solution that they feel best suits the problem and their abilities, rather than enforcing one. It was also shown that they committed errors in their explanations and justifications. Therefore, activities for students should not solely focus on getting the correct procedures and answers. Adding tasks on explanation and justification must be implemented concurrently, as procedural fluency and written adaptive reasoning skills are inextricably linked. Further, it is recommended that teachers should expose students in open-ended problems and allow them to try and justify their own unique solutions irrespective of their mathematical achievement.

Given that only flexibility differs significantly among all components, future researchers may conduct a similar study with a larger sample size in multiple learning modalities. The sample size of this study is limited due to students' internet access under the modular distance learning. A mixed-method research design is also recommended to better comprehend students' solutions and reasoning.

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