Instructional Material for Teaching Fraction

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Elementary learners' struggles with fractions pose a major obstacle to building a strong mathematical foundation. Unlike whole numbers, fractions often lack immediate real-world referents, making their abstract nature especially challenging for young learners (Barmby, 2018). The shift from using concrete tools, such as fraction strips, to engaging with abstract computational tasks intensifies these difficulties, requiring students to develop a deeper conceptual grasp of number relationships (Clarke & Roche, 2020). Individual learner differences, including limitations in working memory or spatial reasoning, further exacerbate comprehension challenges (Fuchs et al., 2013). In addition, widespread misconceptions such as misinterpreting the fraction bar as a subtraction sign frequently result in calculation errors (Bailey et al., 2018).

Research consistently shows that solving fraction-based word problems is particularly problematic for learners (Kenney & Ntow, 2024; López-Martín et al., 2022; Bhatia et al., 2023; Copur-Gencturk & Doleck, 2021). These challenges often stem from difficulties in problem interpretation, translating situations into mathematical expressions, and selecting appropriate operations. Learners also tend to omit units in their

answers, lowering accuracy (Istiqomah & Prabawanto, 2019). Moreover, gaps persist in understanding fundamental fraction concepts, including arithmetic operations and problem-solving strategies (Hariyani et al., 2022).

National and regional assessments mirror these struggles. The Philippines ranked low in mathematics proficiency in PISA 2018 (DepEd – National Report, 2019). While the National Achievement Test (NAT) scores showed modest gains between 2005 and 2013, results still fell short of the 75 percent benchmark. More recently, a district-level assessment in San Jose revealed that 15.8% of Grade 3 learners lacked basic numeracy skills, with fractions identified as a key area of weakness.

These persistent gaps underscore the need to design supplementary instructional materials tailored to fraction learning. Such resources can scaffold conceptual understanding, bridge knowledge gaps, and provide guided practice in solving and creating fraction-based problems. Addressing these challenges not only aligns with the Department of Education's thrust to strengthen numeracy but also highlights the potential of design-based research in producing responsive, evidence-based solutions.

The Impact of Learning Materials in Teaching Fraction

Fractions remain one of the most challenging concepts in mathematics for learners. While many students succeed with whole numbers in early grades, rational numbers often pose significant difficulties (Siegler & Lortie-Forgues, 2017). Proficiency with fractions is not only essential for success in higher-level mathematics but also serves as a foundation for various careers and real-world applications (Fennell & Karp, 2017). The inherent complexity of fractions contributes to learners' struggles, making deep conceptual understanding difficult to achieve

(Kurniawan et al., 2018; Simon et al., 2018). Students with weak foundational knowledge often face difficulties progressing to advanced concepts such as algebra (Loc et al., 2017; Pearn & Stephens, 2015), and poor fraction proficiency has been linked to underperformance in general mathematics and algebra (Ubah & Bansilal, 2018).

Elementary learners, in particular, encounter challenges with foundational concepts, including part-whole relationships, equivalence, and operations like multiplication and division (Barmby, 2019; Jitendra & Griffin, 2020). They often struggle to connect conceptual understanding with procedural fluency, underscoring the need for a curriculum that supports the effective application of fraction knowledge (Staub & Stein, 2018; Cai & Jitendra, 2022). Cognitive and motivational factors also contribute to these challenges. Neglecting embodied approaches can hinder conceptual grasp (Nemirovsky & Ferrara, 2018), while anxiety and low confidence in proportional reasoning further impede learning (Hunting, 2019).

Recent literature highlights the potential of technology-enhanced instruction and contextually relevant examples to address these difficulties. Dynamic visualizations, interactive feedback, and real-world applications can scaffold fraction learning and promote meaningful engagement (Hofer & Reinhold, 2025). These approaches emphasize the importance of targeted instructional materials that bridge gaps in both conceptual understanding and procedural fluency.

Research demonstrates that well-designed instructional materials, particularly gamified and technology-enhanced tools, significantly improve students' fraction learning. Gamified learning strategies, such as fraction board games for third graders, have been shown to boost motivation, encourage strategic thinking, and support discovery-based learning (Khoo

& Wong, 2021; Hsu et al., 2022). Similarly, technology-integrated tools, such as virtual manipulatives with dynamic visual representations, enhance proportional reasoning and explanation skills (Yeo & Park, 2023; Moschkovich, 2019). Interactive computer manipulatives also yield significant gains for students with and without learning disabilities, underscoring technology's role in making fraction concepts more accessible (Ozimek & Jitendra, 2021). However, the success of these approaches depends on careful design, age-appropriate mechanics, equitable access, and students' familiarity with digital tools (Borko & Whitin, 2021; Hsu et al., 2022).

Teacher-developed instructional materials and inquiry-based approaches also support fraction learning by fostering engagement, critical thinking, and problem-solving. Empowering teachers to design activities tailored to their pedagogical styles enables more personalized learning (Borko & Whitin, 2021; Cai & Jitendra, 2022). Culturally responsive tools, such as Mayan fraction bars, can further strengthen engagement by bridging cultural contexts and making learning more meaningful (Rivera & Barroso, 2022). Tangible and interactive learning resources, such as the Fractangi number line, have also been found to foster exploration, enjoyment, and representational understanding (Mpiladeri et al., 2016).

The development and use of instructional materials are crucial for effective teaching and learning. Materials provide essential scaffolding that enables students to construct knowledge independently while supporting teachers in presenting information clearly (Onasanwa & Omosewo, 2011). Incorporating student feedback and teacher consultation ensures that resources are practical, pedagogically sound, and responsive to classroom needs (Moschkovich, 2019; Cai & Jitendra, 2022). Continuous refinement such as adding practice problems, accommodating diverse learning styles,

and simplifying explanations further enhances adaptability and effectiveness (Khoo & Wong, 2021; Borko & Whitin, 2021; Yeo & Park, 2023). Overall, the development of interactive, adaptable, and culturally responsive instructional materials is essential to overcoming challenges in fraction learning and improving students' conceptual understanding and procedural fluency.

Development and Validation of Instructional Material for Teaching Fraction

This study aims to determine the level of acceptability of the developed and validated instructional material for teaching fractions among Grade 5 pupils, focusing on its content, usability, and effectiveness in supporting conceptual understanding and procedural fluency

Methodology

This study employed a design-based research (DBR) approach utilizing the Analysis, Design, Development, Implementation, and Evaluation (ADDIE) model to develop and validate instructional materials for teaching fractions. The research was conducted at Sibalom South District, Schools Division of Antique, Philippines. The participants included 47 Grade 5 pupils, who were administered a diagnostic test on fractions to identify their least mastered competencies, and three mathematics experts who served as validators of the instructional materials.

The data-gathering instruments consisted of two components: (1) a mastery skills test, and (2) an instructional material assessment checklist. The mastery skills test was a 60-item multiple-choice examination prepared with a Table of Specifications to ensure content validity. Following pilot testing, 40 reliable items were retained and used to identify specific learning

gaps in fractions, serving as the foundation for the design of supplementary instructional materials.

Based on the diagnostic results, a 20-item activity sheet was developed, targeting six competencies where learners demonstrated low mastery. The instructional material incorporated problem-solving and problem-creation tasks involving the addition, subtraction, multiplication, and division of fractions and whole numbers. To evaluate the material, the instructional material assessment tool was employed. This checklist measured the quality of the developed instructional material across several dimensions, including content, clarity, accuracy, comprehensiveness, graphical quality, readability, and overall usability, using a 5-point Likert scale.

The research procedure followed three phases:

Phase 1: Preparation and validation of instruments. All instruments were prepared and validated by mathematics experts to ensure reliability and content accuracy. Likewise, the developed instructional materials were reviewed, evaluated, and revised based on expert feedback, ensuring alignment with the identified competencies and adherence to pedagogical standards.

Phase 2: Identification of least mastered competencies. A diagnostic test was administered to identify six least mastered competencies in fractions. Based on the results, instructional materials were developed to enhance pupils' problem-solving and problem-creation skills specifically targeting these competencies.

Phase 3: Experimental teaching. A pretest was administered to determine the pupils' initial mastery levels in fractions prior to the intervention. The pupils were then taught using the developed instructional materials, which included activity sheets and supplementary videos, over a

two-month period. Following the intervention, a posttest was administered to measure their mastery levels after using the instructional materials.

Data were analyzed using the Statistical Package for the Social Sciences (SPSS) version 22. The mean was computed to determine pupils' scores in the diagnostic test, pretest, and posttest, as well as to assess the acceptability of the instructional materials. To determine whether there were significant differences between the pretest and posttest results, a t-test for dependent samples was employed.

Ethical considerations were strictly observed throughout the study. Coordination with school authorities was secured, and informed consent letters were distributed to both pupils and their guardians. Participants were made aware of the purpose of the study, the voluntary nature of their participation, and the confidentiality of their responses. Informed consent was obtained prior to administering the instruments, ensuring that all participants fully understood their rights and the study procedures.

Findings

Table 1 presents the least mastered mathematics competencies of Grade 5 pupils, specifically on operations involving fractions. A total of 47 pupils were assessed across six competencies, yielding an overall mean score of 27.81 with a standard deviation of 3.79, which is interpreted as "high." Among the competencies, the highest mean score was recorded in "Solves routine and non-routine problems involving addition and subtraction of fractions using appropriate problem-solving strategies and tools" (M = 6.26). This indicates that, while addition and subtraction of fractions continue to pose challenges, pupils demonstrated comparatively better performance in applying problem-solving strategies and tools. Nonetheless, consistent practice and scaffolding remain necessary to

strengthen their proficiency.

On the other hand, competencies with lower mean scores highlight areas where pupils experience greater difficulty, underscoring the need for targeted instructional interventions. These results provide a clear basis for developing supplementary instructional materials to address pupils' learning gaps in fraction operations.

Table 1

Least mastered competencies of Grade 5 pupils

Most Essential Learning Competencies	Mean Score
1. Solves routine and non-routine problems involving addition and/or subtraction of fractions using appropriate problem-solving strategies and tools.	6.26
2. Creates problems (with reasonable answers) involving addition and/or subtraction of fractions using appropriate problem-solving strategies.	4.7
3. Solves routine and non-routine problems involving multiplication without or with addition and subtraction of fractions and whole numbers using appropriate problem-solving strategies and tools.	5.28
4. Creates problems with (reasonable answers) involving multiplication of fractions.	4.38
5. Solves routine or non-routine problems involving division without or with any of the other operations of fractions and whole numbers using appropriate problem- solving strategies and tools.	4.43
6. Creates problems (with reasonable answers) involving division or any of the other operations of fractions and whole numbers.	2.83
Over All Mean Score	**27.81**
Description	**High**
SD	**3.79**

Note: 32.01-40.00 Very High; 24.01- 32.00 High; 16.01-24.00 Moderate; 8.01-16.00 Low; 0.00-8.00 Very Low

In contrast, the competency "Creates problems (with reasonable answers) involving addition and/or subtraction of fractions using appropriate problem-solving strategies" (M = 4.70) received a lower mean score. This indicates that pupils found problem creation more difficult than

problem solving. The challenge may be attributed to their limited exposure and practice in formulating mathematical problems, reflecting difficulties in the creative aspect of mathematics learning (Loc et al., 2017; Hariyani et al., 2022). This suggests the need for instructional strategies that encourage creative thinking and problem construction.

For multiplication, the competency "Solves routine and non-routine problems involving multiplication without or with addition and subtraction of fractions and whole numbers using appropriate problem-solving strategies and tools" (M = 5.28) reflected moderate proficiency. Pupils demonstrated slightly better understanding when solving multiplication problems than when creating them, but still required improved strategies for integrating multiple operations. Meanwhile, "Creates problems (with reasonable answers) involving multiplication of fractions" (M = 4.38) showed that pupils had difficulty formulating multiplication problems. This highlights gaps in applying conceptual understanding to higher-order tasks, underscoring the need for instructional practices that strengthen reasoning and creativity.

The competency "Solves routine or non-routine problems involving division without or with any of the other operations of fractions and whole numbers using appropriate problem-solving strategies and tools" (M = 4.43) revealed moderate challenges. While pupils could attempt division problems, their application of strategies and tools remained weak. Finally, the lowest mean score was obtained in "Creates problems (with reasonable answers) involving division or any of the other operations of fractions and whole numbers" (M = 2.83). This was identified as the least mastered competency, suggesting that pupils struggled significantly with formulating division problems. These findings emphasize the need for targeted interventions, particularly instructional practices that foster problem

creation, strengthen conceptual understanding of division with fractions, and promote higher-order thinking skills.

The needs assessment thus served as the foundation for the iterative design and refinement of the instructional materials. The diagnostic test revealed persistent challenges in both problem-solving tasks and problem creation across the four fundamental operations. To address these gaps, instructional materials were specifically developed to target the identified areas of difficulty.

The core intervention took the form of a 20-item activity sheet designed to enhance pupils' mastery of problem-solving and problem-creation in fractions. The activity sheet emphasized both solving and constructing problems involving addition, subtraction, multiplication, and division of fractions and whole numbers. Its primary objective was to equip pupils with effective strategies and tools for solving both routine and non-routine problems, while also fostering creativity and higher-order thinking in problem formulation.

The activity sheet was carefully structured around the least mastered competencies in fractions, which included:

- Solves routine and non-routine problems involving addition and subtraction of fractions using appropriate problem-solving strategies and tools (4 items)
- Creates problems (with reasonable answers) involving addition and subtraction of fractions using appropriate problem-solving strategies (3 items)
- Solves routine and non-routine problems involving multiplication without or with addition and subtraction of fractions and whole numbers using appropriate problem-solving strategies and tools (4 items)
- Creates problems (with reasonable answers) involving multiplication of fractions (3 items)
- Solves routine or non-routine problems involving division without or with any of the other operations of fractions and

whole numbers using appropriate problem-solving strategies and tools (3 items)

- Creates problems (with reasonable answers) involving division or any of the other operations of fractions and whole numbers (3 items)

By directly targeting these least-mastered competencies, the instructional material sought to strengthen pupils' procedural fluency and deepen their conceptual understanding, while also enhancing their problem-posing skills. In doing so, it aimed to bridge critical learning gaps in fractions and promote higher-order mathematical thinking

Table 2

Mastery level of Grade 5 pupils in solving fractions before and after the utilization of the validated instructional materials

Test	Mean Score	Description	SD
Pre-test	9.94	Moderate	2.00
Post-Test	13.06	High	2.12

Note: 16.01-20.00 Very High; 12.01- 16.00 High; 8.01-12.00 Moderate; 4.01-8.00 Low; 0.00-4.00 Very Low

Table 2 presents the baseline assessment of Grade 5 pupils' proficiency in fractions prior to the implementation of the validated instructional materials. The results revealed a mean mastery level of 9.94 with a standard deviation of 2.00, interpreted as "moderate." This indicates that pupils demonstrated a moderate ability to solve addition and subtraction problems with fractions, reflecting some competence in applying problem-solving strategies and tools effectively. However, they still required additional practice and guidance to strengthen these skills. In contrast, pupils exhibited greater difficulty in generating their own problems for

these operations, suggesting challenges in the creative dimension of mathematics, likely due to limited exposure and practice in problem formulation.

The findings further indicate that pupils struggled with multiplication and division problems, both in solving and creating them. Difficulties were particularly evident when applying conceptual knowledge to problem formulation, underscoring gaps in higher-order thinking skills. Division tasks, in particular, revealed limited proficiency in the application of strategies and tools, highlighting the need for instructional interventions that foster problem construction and deepen conceptual understanding.

Following the integration of the validated instructional materials, pupils' mastery showed notable improvement. The mean score increased to 13.06 with a standard deviation of 2.12, classified as "high." This suggests that pupils' ability to solve fraction problems improved considerably, reflecting a positive shift in their capacity for creative application of mathematical concepts. Progress was particularly evident in strategies for multiplication, although many pupils continued to demonstrate limited proficiency in integrating multiple operations involving fractions. These results point to the necessity of sustained, targeted instruction that builds progressively on existing knowledge. Overall, the findings indicate gradual improvement in solving complex fraction problems, demonstrating enhanced conceptual understanding, problem-solving competence, and creative application of mathematical knowledge.

Table 3 presents the t-test results, illustrating the significant difference in Grade 5 pupils' mastery levels before and after the utilization of the validated instructional materials.

Table 3

T-test results on the differences in the level of mastery before and after the utilization of the validated instructional materials

Tests	Mean	t-value	df	p	Interpretation
Pretest	9.94	-15.395	46	.000*	Significant
Post-test	13.06				

**p.< .05, significant*

The results revealed a significant difference in the mastery level of Grade 5 pupils in solving fractions after using the validated instructional materials, $t(46) = -15.395$, $p < .05$. This indicates that pupils who utilized the instructional materials demonstrated a significantly higher level of mastery in fraction problem-solving. These findings suggest that the validated instructional materials effectively enhanced pupils' understanding and ability to solve fraction problems.

However, the results also indicated that while most competencies showed significant improvement, one area did not, highlighting the need for continuous assessment and potential curriculum adjustments to ensure comprehensive mastery of all fraction competencies. These outcomes reinforce the claim that the developed instructional materials successfully addressed the identified learning gaps, providing appropriate strategies and tools to strengthen fraction skills (Shin et al., 2022).

Furthermore, the study demonstrates the potential of targeted, design-based instructional materials to effectively support Filipino elementary pupils in overcoming specific learning difficulties. Table 4 presents the level of acceptability of the instructional materials, as evaluated by three mathematics experts across multiple criteria, including learning content, quality, comprehensiveness, conciseness, clarity, accuracy, graphical completeness, and readability.

Table 4

Level of acceptability of the developed instructional material

Indicators	Mean	Interpretation
Learning Content	3.5	Acceptable
Quality	3.92	Acceptable
Comprehensiveness	4.17	Acceptable
Conciseness	3.92	Acceptable
Clarity	4.18	Acceptable
Accuracy	3.67	Acceptable
Graphical Completeness	4.00	Acceptable
Readability	4.58	Very Acceptable
Overall Mean	**3.99**	**Acceptable**

The results indicated that the overall acceptability of the instructional materials was rated as acceptable, with a mean score of 3.99. This demonstrates that the materials were well-received by evaluators and are likely effective in supporting the teaching and learning of fractions among Grade 5 pupils.

Specifically, the Learning Content received a mean score of 3.50, classified as high, suggesting that the materials comprehensively covered fraction-related topics and contributed to a robust learning experience. The Quality of the instructional materials was similarly rated high, with a mean score of 3.92, indicating that the content was valuable and effectively supported student learning.

In terms of Comprehensiveness, the materials achieved a mean score of 4.17, highlighting thorough coverage of fraction concepts and promoting deeper understanding. Conciseness received a mean score of 3.92, showing that the materials presented information effectively without unnecessary complexity, thereby enhancing learning efficiency. Clarity was rated high at 4.18, reflecting straightforward and easily understandable presentation of concepts. Accuracy scored 3.67, demonstrating the precision of the fraction-related content.

The Graphical Completeness of the materials was rated high (M = 4.00), indicating effective integration of visual elements that supported comprehension. Finally, Readability received the highest rating, with a very high mean of 4.58, suggesting that the materials were exceptionally clear and accessible to learners. These results collectively affirm that the instructional materials are pedagogically sound, user-friendly, and capable of enhancing students' mastery of fraction competencies.

Conclusion

The results indicate that well-designed instructional materials can significantly enhance pupils' understanding and application of fraction concepts. At the same time, the findings underscore the importance of continuous refinement to ensure that all areas of fraction learning are comprehensively addressed. This highlights the need for a targeted and adaptive approach in developing instructional materials, enabling students not only to improve in specific competencies but also to achieve well-rounded mastery of fractions. Consequently, sustained efforts in curriculum development and instructional innovation are recommended to fully support pupils' mathematical growth.

The positive evaluation across multiple dimensions of instructional quality validates both the design and content of the materials. It also emphasizes the critical role of readability, clarity, and engaging presentation in facilitating learning. These results suggest that instructional resources should remain accessible, visually appealing, and learner-centered to enhance students' experiences and outcomes in mathematics. Future materials should continue to prioritize clarity, accuracy, and engaging content while incorporating regular assessment and feedback to ensure ongoing improvement and effectiveness.

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