INTERNATIONAL JOURNAL

OF SCIENCE, TECHNOLOGY, ENGINEERING AND MATHEMATICS

VOLUME 2 ISSUE 2 • JUNE 2022

ISSN 2799-1601 (Print) • 2799-161X (Online)

For publication concerns, contact the publisher at ijstem@iiari.org.

ISSN 2799-1601 (Print)
2799-161X (Online)

Published by:
Institute of Industry and Academic Research Incorporated
South Spring Village, Bukal Sur
Candelaria, Quezon, Philippines
Postal Code 4323
Contact Numbers: (042) 785-0694 • (+63) 916 387 3537
Visit the website https://iiari.org

INTERNATIONAL JOURNAL OF SCIENCE, TECHNOLOGY, ENGINEERING AND MATHEMATICS

ISSN 2799-1601 (PRINT) 2799-161X (ONLINE)

Volume 2 Issue 2 | June 2022

ISSN 2799-1601 (Print)

2799-161X (Online)

This journal is published quarterly every March, June, September and December.

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International Journal of Science, Technology, Engineering and Mathematics
Volume 2 Issue 2 June 2022
DOI: https://doi.org/10.53378/352885

Problem-Based Learning Strategies and Critical Thinking Skills Among Pre-Service Teachers

[1]Patricia Nicole F. Benedicto & [2]Rose R. Andrade

Abstract

Mathematical underachievement among students was not only a source of concern in the Philippines, but has now spread throughout the world. Low critical thinking skill among Filipino students is one of the causes contributing to the country's poor performance in mathematics. Students' lack of critical thinking abilities may be due to teachers' knowledge and expertise. To explore the critical thinking skills of pre-service teachers through the use of problem-based learning strategies is the main objective of the study. Quasi-experimental with a counterbalanced design was employed, presenting two problem-based learning strategies namely authentic strategy and non-linear strategy to two groups of teacher candidates. The results showed that students' critical thinking skills in evaluating arguments and drawing conclusions are lacking. There is also a substantial difference in critical thinking skills between students in groups 1 and 2, except for the capacity to recognize assumptions. The pre-service teachers' critical thinking skills do not differ significantly by sequence. The study results indicated that the pre-service teachers' critical thinking skills were still low, particularly in terms of evaluating arguments and drawing conclusions due to inadequate background knowledge and lack of in-depth understanding of the mathematics concepts. The results imply that schools focus more on building strategies to improve and develop students' critical thinking skills in mathematics education. Moreover, the study suggests that further research develop successful techniques for planning effective initiatives to increase critical thinking teaching and learning in higher education and training programs that could help improve the students' critical thinking skills.

Keywords: *Problem-Based Learning Strategy, Authentic Strategy, Non-linear Strategy, Critical thinking skills*

Article History:
Received: *April 14, 2022* | ***Revised***: *April 26, 2022*
Accepted: *May 2, 2022* | ***Published online***: *May 16, 2022*

Suggested Citation:
Benedicto, P.F. & Andrade, R.R. (2022). Problem-Based Learning Strategies and Critical Thinking Skills Among Pre-Service Teachers. *International Journal of Science, Technology, Engineering and Mathematics*, Volume 2 Issue 2, pp. 1 - 28. DOI: https://doi.org/10.53378/352885

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1. Introduction

In today's present circumstances, people are confronted with a plethora of issues in daily lives the require good decisions. The vast majority of decisions are made instinctively, which does not necessitate much deliberation and frequently results in the erroneous option being made (Alban, n.d.). Consequently, it is necessary to foster decision making skills to children through development of critical thinking to make sound decisions (Patterson, 2020). By employing critical thinking skills, individuals would avoid making mistakes and, in general, result to better decisions (Fisher, 2011). Thus, World Economic Forum listed critical thinking as one of the 21st-century skills that learners must acquire, which could eventually enhance students' academic performance (Rayhanul, 2015).

In the academic setting, higher level performance is associated with critical thinking (Wulandari et al., 2021). Critical thinking is the ability to think logically and realistically, which aids in the ability to successfully grasp and solve problems in an appropriate manner. According to Chikiwa and Schäfer (2018), critical thinking can assist learners in discovering new and more effective approaches to tackle problem situations. Because mathematics has a structured form and is extremely easy to understand in terms of its function, students can develop their rational, logical, and critical thinking abilities through mathematics learning (Aizikovitsh & Amit, 2011).

Muhlisin et al. (2016) argue that many students lack critical thinking skills, and this is linked to the traditional teaching style used in the classroom. As it is not a simple task to shift the mindset and behavior of present teachers in order for them to learn to think critically (As'ari et al., 2017), educating future teachers is far more strategic than teaching current teachers (Prahmana et al., 2012). This implies that it is a better choice to train future teachers to become critical thinkers (As'ari et al., 2017). Rusmansyah et al. (2019) affirmed that one key factor contributing to the enhancement of pre-service teachers' critical thinking skills is problem-based learning (PBL). It is a powerful pedagogical strategy that provides opportunities for students to learn how to think critically. In information-rich environments, PBL challenges students to solve real-world problems where they should establish a solution that leads to the most effective experience, such as methods, processes, and epistemology (Yazar, 2015).

The K-12 Basic Education Mathematics Curriculum aims to improve students' critical thinking and problem-solving skills (DepEd, 2012). Despite the curriculum's emphasis, students' mathematics performances are still low. In the 2017-2018 National Achievement Test (NAT), mathematics had the lowest mean percentage score of 35.34, and it also had the lowest mean percentage score in critical thinking with 32.42. According to Lugtu (2018), the absence of critical thinking skills among Filipino students has been a problem for generations. Furthermore, Marquez (2017) stated that the primary contributing element to this problem is the stress placed on rote memorization, wherein recitations and tests simply serve to strengthen the memorization capacity of the students in the first place. It has become worse because of the increased use of smartphones and technology. In addition, technology hinders the student's ability to address the problems because majority of the solutions are available on the internet (Rodzalan & Saat, 2015).

When it comes to problem-solving, one of the reasons why Filipino students struggle is because they frequently solve the same problems as their teachers do (Salangsang & Subia, 2020). It is possible that some students have a strong understanding of math principles, but when they are asked to apply those concepts in the real world and write them down, they find it more challenging (Angateeah, 2017). Even mathematics pre-service teachers have had difficulty understanding the stages of a problem because they have failed to classify the problems appropriately (Dimasindel, 2017). According to Saputra et al. (2019), by presenting problems to students, the learning process allows them to improve their critical thinking skills by trying to solve them. Exposure to authentic non-routine, and ill-structured problems serve as avenues to hone the critical thinking skills of the students (Apriliana et al., 2019; Bingolbali, 2011; Romanoff, 2019). Therefore, critical thinking skills could be developed using several types of problems.

The research conducted by Arviana et al. (2018) and Yuliati et al. (2018) are deemed parallel to this investigation. While the primary purpose of the current study is to explore the effects of PBL in the critical thinking skills of teacher-candidates, the two previous studies were conducted in another country, and the majority of the respondents were in middle school. The same research paradigm would be explored and applied at one state university in the Philippines. Thus, the current study aims to investigate the effects of PBL on pre-service teachers' critical thinking skills. Further, its specific goal is to assess the critical thinking skills of a group of students exposed to the first sequence (Non-Linear – Authentic) and the second sequence (Authentic-Non-Linear) in terms of their ability to recognize assumptions, evaluate arguments, and draw conclusions. It also seeks answers to the following hypothesis:

Ho1: There is no significant difference in the critical thinking skills based on their group and sequence as to non-linear-authentic and authentic-non-linear.

2. Literature Review

2.1. Problem-Based Learning

Padmavathy (2013) asserts PBL as the most effective way to teach mathematics. By adopting the PBL method, teachers can create engaging mathematics lessons. The business world requires many creative thinkers, critical decision-makers, and problem solvers who make creative decisions. A problem-based learning instructional strategy positively affects content knowledge, creating increased learning opportunities to learn content and stimulating the students' enthusiasm, interest, participation, and motivation. This positively motivates the learners to have a positive attitude towards math which encourages them to understand math better and leads to better retention of math facts. It provided the audience with a predictable and desired outcome. Similarly, Loyens et al. (2015) provide shreds of proof of PBL's effectiveness. He held three groups of students as they were exposed to three different methods. The PBL group's performance was more likely to demonstrate on-task performance, outperforming those students exposed to the lecture-based and self-study groups. It was further supported by Tupas (2012) that whether students solved the problem as a group or individually, they improved significantly. It leads them to be creative enough in finding the best solution, which is the essence of mathematics instruction.

Ibrahim et al. (2018) describe PBL as a useful learning strategy. Based on students' answers, the effectiveness of the PBL was found to increase through experience. Though programs based on PBL play a significant role in acquiring information and soft skills, it is crucial to choose the intended program to capture and evaluate the effects of PBL. PBL is active in soft skills enhancement; therefore, tutors' training to master essential basics in processes, skills, and attitudes may be required to deal with this method effectively. Rokhmawati et al. (2016) inferred that PBL models can be applied to strengthen students' problem-solving skills. The problem presented in the learning process illustrates the real-world problems that people face on a daily basis. Implementation of the PBL model may also increase students' self-efficacy (Maulidia et al., 2020), cognitive abilities (Khoiriyah & Husamah, 2018) and problem-solving skills.

According to Asad et al. (2015), teaching through PBL enhanced students' ability to solve problems and think critically, with 60% of first-year students and 71% of second-year students agreeing that these sessions improved their ability to learn knowledge and use it to solve multiple-choice questions (MCQs). They discovered that their ability to learn how to read nonverbal cues, demonstrate a higher level of intrinsic motivation, and develop creative precepts for clinical reasoning had improved. The students who took part in projects or collaboratively worked together found their projects to be quite successful. In another study by Andini and Hobri (2017), the PBL was orientated in a Lesson Study for Learning Community (LSLC), hence increasing student participation in meaningful learning. Students' tasks include reviewing and presenting problems, strategizing, putting the plan into action, and examining and evaluating the results, with a high average value activity for each subsequent stage. Similarly, the study conducted by Arviana et al. (2018) showed PBL’s impact on students' ability to think critically. Students were asked to provide a “yes or no” response throughout the learning process, as well as a “why and how” response. The pupils were urged to be able to take or validate a position on a subject. Likewise, Yuliati et al. (2018) stressed that the application of authentic problem-based learning to physics learning enhances students' critical thinking capabilities. Therefore, problems dealing with real-world issues are preferred to be used to hone students’ critical thinking skills.

Non-Linear. Students learn in a variety of ways and receive a wide range of perspectives, thoughts, ideas, and solutions as a result. In this way, it is an issue that may be approached in a variety of ways. Apino and Retnawati (2017) discovered that using more than one answer is an efficient strategy for students to practice their arithmetic thinking skills because it requires them to think in a creative manner. Furthermore, as students develop their ability to think creatively, they will be able to discover new ways for dealing with complex challenges. According to Guberman and Leikin (2012), solving tasks that permit multiple solutions enhance teachers' problem-solving skills. The discovery of various solutions is part of pedagogical awareness of material, a significant indicator of students' academic achievements in mathematics (Baumert et al., 2010).

Arikan (2016) highlighted that prospective teachers state that they sustain their recollection of the formulas, rules, and knowledge they once forgot in conjunction with problem-solving. They felt that using multiple approaches to problem-solving would improve their academic knowledge and become experts in their fields. When teachers look for new educational solutions in the classroom and when they share ideas outside of the classroom, students' views about mathematics lessons will be more favorable. Further, Bingolbali (2011) asserted that students' creativity and critical thinking skills are enhanced as a result of attempting to solve problems in a variety of methods. Developing multiple solutions positively affects students’ enjoyment and adverse effects on their boredom (Schukajlow & Rakoczy, 2016).

Authentic. In education, the word "authentic" is widely used, mainly without much meaning. Some scholars have attempted to establish the authenticity elements and aspects to understand authentic contexts (Wang et al., 2012). Based on Roach (2018), the term authentic learning refers to learning immersed in an environment that aligns learning goals with activities, content, and context in the real world. Authentic learning is when students use their own knowledge, experience, and resources to learn about new ideas and techniques.

Through real-life connections, learners can improve their knowledge and achievement in mathematics (Karakoc & Alacaci, 2015). Various research stated that the more motivated students are, the more likely they are to perform well in mathematics (Muijs & Reynolds, 2011). Further, Apriliana et al. (2019) stated that using authentic problems prompt students to solve the task by finding possible solutions and appropriate strategies. With consistent use of real-world problems, it will ignite students' mind and knowledge in doing problem-solving tasks.

2.2. Critical Thinking Skills

Critical thinking is a cognitive activity related to the use of the mind (Padmanabha, 2018) in order to interpret, assess, evaluate, and explain wrong information (Saputra et al., 2019). The ability to think critically indicates the use of mental processes such as attention, categorization, selection, and judgment (Kumar & James, 2015); concepts such as cognitive and meta-cognitive skills, practices and abilities, dispositions and character, logic, and reflection (Ennis, 2011) and skills such as analyzing, synthesizing, evaluating, and summarizing (Dwyer et al., 2014). As a learning method, critical thinking emphasizes the agreement or disagreement with facts, judge reality, and modify misinformation to generate new ideas (Florea & Hurjui, 2015). However, apart from a lack of capacity, many individuals who can develop more efficient analytical thinking can be prevented from doing so for different reasons. Personal and emotional or affective factors, in particular, may create barriers (Cottrell, 2011).

Critical thinking is a higher-order thinking skill that can be considered reflective thinking (Apriliana et al., 2019). According to Nuriadin et al. (2015), someone who uses reflective thinking will have the ability to identify problems, choose alternative solutions, analyze problems and evaluate solutions, and conclude and decide the best solution to the problem given. As Jensen (2011) has mentioned, critical thinking requires an efficient and effective mental process to follow appropriate and accurate knowledge. Wijaya (2011) discussed the task of examining thoughts or ideas in a more specific way, separating them clearly, selecting and recognizing them as well as pursuing further research and enhancing them in a more effective way.

Mustaji (2012) believes that critical thinking is rooted in making decisions about what to believe or do, which is synonymous to decision-making, strategic planning, and problem-solving (Husnaeni, 2016). It is a method of increasing knowledge and intellect by contrasting several current and potential issues in order to arrive at a conclusion and a solution. Instead of outright acceptance of various concepts, critical thinking entails deep reasoning and analysis (Fahim & Pezeshki, 2012). This means that people's opinions and suggestions about a phenomenon cannot be believed entirely until they go through a structured and rational process of discovering the facts. Learning to think critically is essential for applying critical thinking in school settings because people who think critically are able to recognize and correct common logic errors as well as understand and connect logical connections between concepts. Critical thinkers can also construct and test arguments, recognize and correct common logical errors as well as solve problems systematically (Chukwuyenum, 2013). Efforts to improve critical thinking in mathematics education have become the focus of global mathematics curriculum development. Critical thinking promotes originality and autonomous thinking by encouraging students to apply critical thinking skills in their regular activities and assignments (Firdaus et al., 2015).

Recognize Assumptions. Ekstrom (2021) defines assumption as an unexamined belief because the conclusion or inferences begin with assumptions that have not been critically analyzed and tested, and assumptions are what we presume without evidence. Assumptions are considered accurate based on a lack of evidence. It is easy to assume all information presented is true, even though not all was provided. Recognizing assumptions allows the identification of the factual evidence presented and how relevant it is. Identifying assumptions helps discover pertinent information, highlights issues and gives a better overview of issues (Davis, 2019). Several proposed assumptions follow each statement. The learner must determine whether these assumptions can be taken for granted, whether they are justifiable, and whether they are unjustifiable (Kumar & James, 2015). The stronger the assumptions, the stronger the thinking **(**Meegan, 2012).

According to Egan (2016), when people learn that something is based on assumptions, they tend to take it for granted that it is accurate. These are considered part of their belief system. They feel that their values and assumptions are correct, and they utilize these beliefs to form their perception of the world. It is from these assumptions that beliefs and conclusions are formed, which may be logical or irrational depending on whether there is evidence to support the assumptions. The goal is for the difference to be recognized and appreciated. Inferences are built on the foundation of assumptions. In order to make sense of what is going on around as rapidly as possible, people generate inferences. Assumptions and the inferences that result from them permeate every aspect of our life.

***Evaluate Arguments*.** The core component of critical thinking is argument. Arguments are defined as assertions meant to convince someone to do something or believe something. Understanding arguments is crucial in developing critical thinking skills—analyzing an assertion and doing so accurately. Arguments can create agreement or disagreement with the information given (Kuhn, 2010) but the validity of arguments requires critical thinking skills (Chen, 2014). Arguments can be wrong, and this part of the RED model teaches how to recognize the tendency to discover and consider proof that confirms prior beliefs (Davis, 2019). It is desirable to differentiate strong and weak arguments when making decisions about important issues. An argument is weak if it is not explicitly connected with the problem (Kumar & James, 2015). Students must back up their claims with logical arguments that are supported by real examples or facts that are in opposition to their claims (Indrawatiningsih, 2018). It must be relevant and directly related to the problem for a claim to be strong.

Draw Conclusions. According to Bahatheg (2019), inference is described as the ability or mental ability to use what was known to draw a vague or missing conclusion from experience and facts. It is also known as executive cognitive ability, which helps learners demonstrate the degrees of accuracy or inaccuracy of a particular outcome using their experiences and all available knowledge, or clarify a missing component of its aspects based on its relativity to the information. It deals with the conclusion that logically follows from the available evidence. Individuals who hold this skill do not make inadequate generalizations beyond the evidence. People with good discernment are typically viewed as having good judgment because they make good decisions (Davis, 2019). Relatively, an inference is a conclusion that can be drawn from some observed or assumed facts by an individual. Several possible inferences follow each statement of fact. Conclusions are drawn from the stated facts (Kumar & James, 2015).

3. Methodology

a. Research Design

Quasi-experimental with counterbalanced design was used to investigate the critical thinking skills of pre-service teachers in terms of recognizing assumptions, evaluating arguments, and drawing conclusions through problem-based learning strategies. Similarly, this method was used since the aim was to determine whether there was a significant difference in critical thinking skills according to their group and sequence as to non-linear – authentic strategy and authentic-non-linear strategy. A different sequence of interventions is administered to each group than to the others followed by an observation of the outcome (Singh, 2021).

3.2. Respondents of the Study

Thirty pre-service teachers from one state university in the Philippines took part in this study during the academic year 2020-2021, which was conducted towards the end of the school year. Purposive sampling with matching was used to select the respondents for this study because only students who took geometry were considered for inclusion. When it comes to geometry subjects, non-linear and authentic questions are more prevalent, and these challenges serve as the foundation for developing, modifying, and validating research instruments.

3.3. Instrumentation and Data Collection

The details of the purpose and development of the instruments are the following:

Assessment of Critical Thinking Skills as exposed to Non-Linear and Authentic Strategies. This consists of six questions, 3-item test for each strategy consisting of typical word problems with additional questions designed to foster critical thinking. This is based on Usta (2020) and was evaluated by four mathematics teachers to ensure content validity.

Scoring Rubric for Critical Thinking Skills. As part of the evaluation of critical thinking skills, students were exposed to non-linear and authentic strategy. This was used to evaluate the students' responses to the assessment of critical thinking skills. There are four mathematics teachers and one English teacher who validated this rubric, which ensured that the rubrics were linked with the essential learning competency.

Data Collection. The researcher divided the class into two groups. Each group was exposed to non-linear strategy and authentic strategy. The first group was exposed to the non-linear – authentic strategy. The questions given to the first group could be solved in multiple ways. The respondents were given 15 minutes to accomplish each question. On the other hand, second group was exposed to the Authentic – Non-linear strategy. This implies that while the first group was answering the first set, the second group was provided three questions utilizing Authentic Strategy. The students had to seek answers to the questions using real-world or relevant problems that are meaningful to respondents. On the second session, the first group and second were exposed to Authentic Strategy and Non-linear strategies, respectively. This was the time that both groups had interchanged answering the question sets that were provided to them during the first session. Afterwards, the researcher guided the students by providing some questions to assess the students' critical thinking skills. Lastly, the last 5 minutes in each session were used to finalize and compile answers before the submission. After students completed the test, several students in each group underwent an interview. The researcher used the interview to clarify the students' answers particularly to complete the students' responses to Problem-Based Learning Strategies, which could not be found in the assessment. The assessment answered through the implementation of strategies was interpreted using the researcher-made critical thinking scoring rubric.

Data Analysis. In analyzing the critical thinking skills of the students, frequency and percentage were used. To determine whether there is no significant difference on the critical thinking skills of the students, paired t-test was used. Furthermore, independent t-test were applied to identify if significant difference exists on the critical thinking skills of the students according to their sequence.

3.4 Ethical Consideration

The study maintained the privacy of the results and the personal details of the respondents. The researcher and the thesis advisor were responsible for the outcomes of the data in the sample test. The names of the respondents were kept highly confidential all throughout the completion and publication of the study.

4. Findings and Discussions

Table 1
Level of Critical Thinking Skills of the Students Exposed in the First Sequence

Score Range	Non-Linear		Authentic		Average		Interpretation
	f	%	f	%	f	%	
Recognize Assumption							
3.50 - 4.00	1	6.7	1	6.7	1	6.7	Advanced
2.50 - 3.49	3	20	9	60	7	46.7	Proficient
1.50 - 2.49	7	46.7	2	13.3	5	33.3	Developing
1.00 - 1.49	4	26.7	3	20	2	13.3	Beginning
Evaluate Arguments							
3.50 - 4.00	2	13.3	1	6.7	-	-	Advanced
2.50 - 3.49	4	26.7	2	13.3	4	26.7	Proficient
1.50 - 2.49	8	53.3	8	53.3	10	66.7	Developing
1.00 - 1.49	1	6.7	4	26.7	1	6.7	Beginning
Draw Conclusions							
3.50 - 4.00	1	6.7	-	-	-	-	Advanced
2.50 - 3.49	2	13.3	-	-	2	13.3	Proficient
1.50 - 2.49	6	40	5	33.3	8	53.3	Developing
1.00 - 1.49	6	40	10	66.7	5	33.3	Beginning

Table 1 shows the level of critical thinking of the students exposed in the first sequence through recognizing assumption, evaluating arguments and drawing conclusions as indicators.

In terms of recognizing assumption, there are more students at the proficient level (46.7%). This shows that students were able to identify at least one correct mathematical concept or fact used to solve the question. It is possible that time constraint was the driving force for the creation of one mathematical concept wherein students did not have enough time to double-check and consider an alternative concept to the one presented in the question. Furthermore, some students lacked the necessary understanding to use suitable terminology, which resulted to inability to correctly identify some of the ideas discussed. As a result, only common concepts were determined by pre-service teachers. According to Caviola (2017), the presence of a time constraint can have an adverse effect on performance in any arithmetic or problem-solving circumstance. As a result of time constraints, students may either become more involved in the work or may choose an improper notion to solve the task. On the other hand, it also displays that using authentic strategy, the majority of the students are at the proficient level with 60%. This implies that the students were able to identify two or three mathematical concept/s or fact/s used to solve the problem. Some students have experienced problems similar to those presented in this strategy, making them more adept at identifying them. Mathematics proficiency, or the ability to think about mathematical problems, is contingent upon familiarity with mathematical concepts. Student engagement in a range of problem-solving scenarios is influenced by the amount and breadth of relevant knowledge a student can recall, as well as the breadth and depth of ideas a student knows (Lindquist et al., 2019).

It is evident that most of the students perform better in authentic strategy in terms of recognizing assumptions. According to Carvalho et al. (2015), implementing a solution based on real-life situations is a powerful strategy for improving problem-solving ability in the sciences classroom. As a result, this strategy enables students to develop their critical thinking skills, particularly in assessing and identifying the concepts related to solving complicated real-world problems. Furthermore, it is also apparent that one student belongs in the advanced level in both strategies. This only means that she was able to identify clearly, accurately, and appropriately more than three key concepts or facts that were used to solve the problem. It was also noted that Student 1 is one of the outstanding students in their class, especially in mathematics subjects. According to Lepasana (2018), students who excel in mathematics typically possess good critical thinking abilities to identify mathematical concepts and solve complex mathematical problems. Likewise, she was also a former student from another university program, in which she has already taken higher mathematics subjects. According to Rogers (2013), mathematical exposure has a more significant influence on mathematics achievement and develop mathematical critical thinking skills.

In terms of evaluating arguments, the majority of students in the non-linear and authentic strategies were classified as developing, with 53.3% each. This shows students were able to justify one of the results of the procedures and seldom explain their answers. As explained by Ergen (2020), this could be because they did not identify all of the fundamental concepts that were required for solving the questions in order to provide a complete explanation of the solutions to the questions. Nevertheless, some students belong to the advanced level. Generally, most of the students exposed to the first sequence in terms of evaluating arguments are on the developing level with 66.7%, which means that the students have a fair performance in conducting a systematic and comprehensive examination of given evidence and arguments and in justifying and explaining how they arrived at their answers.

In non-linear strategy, there are two students at the advanced level: Student 5 and Student 11. This implies that they were able to justify more than three results or procedures and explain their answer thoroughly. While they have reached the proficient level in recognizing assumptions, they still demonstrate exceptional performance when it comes to explaining the process by which they arrived at their answers. After they are exposed to authentic strategy, they fall into the proficient category. Meanwhile, in authentic strategy, only one student reached the advanced level, which is Student 1. This means that Student 1 was able to justify more than three results or procedures and thoroughly explain their answer. It is also observed that Student 1 is the only student who belongs to the advanced level in terms of recognizing assumptions in both strategies. According to Apostol (2017), students who are more accurate in their use of mathematical concepts and ideas are better able to carry out a procedure entirely and explain the answer adequately. For this reason, Student 1 exhibits an excellent performance in explaining the procedure of the solution she was provided. Nonetheless, she was at the proficient level when she was exposed to non-linear strategy as a result of having an incorrect answer to one of the problems given.

In terms of drawing conclusions, most of the students under the non-linear strategy are at the beginning and developing levels, with 40.0% each. This shows that most of the students were not able to draw coherent or clear conclusions. It is also noted that the students were able to obtain a relevant but abbreviated or simplified conclusion that is not fully supported. This could be attributed to the fact that some students are doubtful of their answers, which led to having difficulty in presenting and explaining the evidence that would support their conclusion. It means students do not have the best proof to show that their answer is correct.

Conversely, in the authentic strategy, most of the students are at the beginning level, with a percentage of 66.7. This is surprising given the fact that the respondents of this study are mathematics pre-service teachers. It implies that most of the students were not able to draw a coherent and clear conclusion despite the fact that their expertise is mathematics. According to Muhlisin et al. (2016), making deductions is one of the difficulties that needs to be resolved among students. According to the interview, some students say that they could not figure out whether or not their answer was correct when they were faced with complicated and real-world problems. Seemingly, the students somehow performed better when exposed to the non-linear strategy in terms of drawing conclusions. This is consistent with Mabilangan et al. (2011) who found that exposing students to non-routine tasks can help them strengthen their mathematical reasoning skills and comprehend that mathematics is a creative pursuit.

Overall, most of the students exposed to the first sequence in terms of drawing conclusions are at the developing level with a percentage of 53.3, which indicates that the students manifested a fair performance in collecting the information and utilizing it to arrive at logical conclusions and convincing answers. This can be due to the level of their recognizing assumptions, as shown in table 1. According to Egan (2016), assumptions serve as the foundation for inferences. It can be noted that the identified concepts influence the conclusions drawn by the students. Even though the majority of students are proficient in recognizing assumptions, they exhibit a low level of ability in drawing conclusions. The lack of evidence in their arguments and insufficient explanations of their conclusions are discernible in their solutions. This result was supported by Siriwat et al. (2017) that students with low skills developed a conclusion from a single discovery, and most of the conclusions drawn were oversimplified and therefore incomprehensible.

Table 2
Level of Critical Thinking Skills of the Students Exposed in the Second Sequence

Score Range	Non-Linear		Authentic		Average		Interpretation
	f	%	f	%	f	%	
Recognize Assumption							
3.50 - 4.00	1	6.7	-	-	-	-	Advanced
2.50 - 3.49	8	53.3	1	6.7	3	20	Proficient
1.50 - 2.49	5	33.3	7	46.7	11	73.3	Developing
1.00 - 1.49	1	6.7	7	46.7	1	6.7	Beginning
Evaluate Arguments							
3.50 - 4.00	1	6.7	-	-	-	-	Advanced
2.50 - 3.49	3	20	3	20	3	20	Proficient
1.50 - 2.49	7	46.7	9	60	10	66.7	Developing
1.00 - 1.49	4	26.7	3	20	2	13.3	Beginning
Draw Conclusions							
3.50 - 4.00	-	-	-	-	-	-	Advanced
2.50 - 3.49	-	-	-	-	-	-	Proficient
1.50 - 2.49	5	33.3	8	53.3	7	46.7	Developing
1.00 - 1.49	10	66.7	7	46.7	8	53.3	Beginning

Table 2 shows the level of critical thinking of the students exposed in the second sequence through recognizing assumption, evaluating arguments and drawing conclusions as indicators.

In terms of recognizing assumption, most of the students under the authentic strategy are at the proficient level with a percentage of 53.3, which means that the students identify two to three mathematical concepts or facts used to solve the problem. Based on the interview conducted, most of the students had difficulty with the problems presented in the authentic strategy. Nevertheless, they still outperformed the authentic strategy in identifying the mathematical concepts used to solve the problem. They easily connect the ideas in real-life situations due to their prior experiences. According to Welty (2010), experience is the most important factor for students since it generates needs, interests, and motivations to solve problems.

In the non-linear strategy, there are more students who are at the beginning and developing levels, with a percentage of 46.7 each. This entails that the students were not able to identify an appropriate concept that was used to solve the problem. It was also noted that students were able to identify at least one correct mathematical concept or fact that was used to solve the problem. Based on the interview conducted, some students struggled to grasp the problem, and they were confused about concepts they should use to solve the problem. They also do not encounter that kind of problem, so they have difficulty recognizing the concepts they use.

It is apparent that the students perform better when they are exposed to authentic strategy. Cai and Lester (2010) assert that using authentic problems build an inextricable link between problem solving and concept acquisition. In addition, comprehension through concepts and procedures should be practiced to develop the problem-solving skills among students.

Overall, most of the students exposed to the second sequence in terms of recognizing assumptions are at the developing level with a percentage of 73.3, which demonstrates the students have fair performance in identifying the factual evidence presented on a given problem and examining how relevant it is to the problem.

In terms of evaluating arguments, there are more students classified as developing under the authentic and non-linear strategies, with percentages of 46.7 and 60.0, respectively. This shows that the students could justify one of the results or procedures but seldom explain their answers. According to Booth (2011), the concept identified influences their ability to explain and justify their answers and solutions. As seen in table 2, most of the students are at the developing level in terms of recognizing assumptions, and this is why students only justify one of the results or procedures. However, only one student qualifies for the advanced level of the authentic strategy. Student 27 justified key results, procedures and explained the answer thoroughly. Students who can look at and evaluate arguments or claims show that they have critical thinking skills (As'ari, 2017).

In the non-linear strategy, out of 15 students, nine are on the developing level, which means that they were able to justify at least one result or procedure and seldom explain their answers. Apostol (2017) found that students who use math concepts and ideas correctly are more likely to finish a process and explain the answer.

Overall, the majority of the students exposed to the second sequence in terms of evaluating arguments are at the developing level with a percentage of 66.7, which means that the students exhibit fair performance in conducting a systematic and comprehensive examination of given evidence and arguments and in justifying and explaining how they arrived at their answers. Indrawatiningsih (2018) stated that the most crucial main indication of critical thinking ability is students' capacity to formulate arguments for or against material provided to them. Evaluating arguments against known facts is an essential technique for evaluating students' critical thinking abilities, particularly their understanding of arguments. Students must be able to sift through information effectively and not get fixated on arguments/claims made by others.

In terms of drawing conclusion, all of the students under the authentic strategy and non-linear are at the beginning and developing levels with a percentage of 66.7, 33.3, 46.7, and 53.7, respectively. This means students were not able to draw a coherent or clear conclusion. It is also noted that most of the students were able to obtain a relevant but abbreviated or simplified conclusion that is not fully supported. These findings are consistent with the Association of American Colleges and Universities (AACU) (2017), which found that students' ability to form conclusions was a weak point in their study. Similarly, Utami et al. (2019) showed that it was still difficult for students to connect facts, solutions, ideas, and concepts because students lack confidence in interpreting and justifying the solutions because they cannot form valid and supported conclusions. This is contrary to Indriani and Julie (2017), who found that the students' capacity to draw conclusions reached a very high category because they were used to employing deductive thinking, which implies that students used their experiences to develop reasoning, which then became provisions for solving problems.

Generally, all students exposed to the second sequence in terms of drawing conclusions are at the beginning and developing levels with a percentage of 66.7, which means that the students demonstrate a fair performance in collecting the information and utilizing it to arrive at logical conclusions and persuasive responses. According to Visande (2014), the ability to conclude does not stop with assessing the text with knowledge and experience. It also entails judgment based on direct evidence from the text. In the case of the students, they did not conduct a thorough investigation of the given statement. According to the assertions in the test, they have most likely created assumptions rather than reasoned from the information. The students' ability to understand without having to think about it is likely to be common.

Table 3
Difference on the Critical Thinking Skills by Group

Indicators	Nonlinear		Authentic		t	df	Sig. (2-tailed)
	M	SD	M	SD			
Group 1							
Recognize Assumption	2.13	0.78	2.49	0.86	-2.416	14	0.03
Evaluate Arguments	2.33	0.78	1.91	0.71	2.679	14	0.018
Draw Conclusion	1.93	0.86	1.42	0.48	3.286	14	0.005
Group 2							
Recognizing Assumption	2.6	0.63	1.56	0.45	-6.313	14	0
Evaluate Arguments	2.04	0.81	1.89	0.48	-0.847	14	0.411
Draw Conclusion	1.29	0.35	1.49	0.43	1.79	14	0.095

Table 3 shows the difference in the critical thinking skills of the two groups of students.

In terms of group 1, results show that the mean in authentic strategy in terms of recognize assumptions (2.49) is higher than the mean in non-linear strategy (2.13). This implies that the students perform better when exposed to authentic strategy. Carvalho et al. (2015) stated that real-life problem-solving strategies allow students to improve their critical thinking abilities, especially in evaluating and recognizing ideas relevant to addressing complex real-world situations. Accordingly, these data were subjected to statistical analysis, revealing a significant difference (t=-2.416; p=0.030) between the non-linear and authentic strategies. In evaluating arguments, the mean in non-linear strategy (2.33) is higher than the mean in authentic strategy (1.91). This indicates that when students are exposed to the non-linear strategy, they perform better. Students who continuously work on problem-solving (non-routine problems) are more creative and critical in explaining the procedures of the solution and their answers (Maulana et al., 2018). Consequently, these data were subjected to statistical analysis, revealing a significant difference (t=2.679; p=0.018) between the non-linear and authentic strategies. In terms of drawing conclusions, the mean in the non-linear strategy (1.93) is higher than the mean in the authentic strategy (1.42). This reveals that students perform better when exposed to non-linear strategy. According to Pratiwi et al. (2021), students should be used to working on non-routine problems in order to develop their mathematical reasoning abilities and ability to solve problems effectively. As a result, these data were subjected to statistical analysis, which revealed a significant difference (t=3.286; p=0.005) between the non-linear and authentic strategies.

It can be concluded that the respondents perform better when they are exposed to problems where multiple strategies and solutions could be employed. There is a higher chance that their critical thinking skills will be developed. This is consistent with Firdaus et al. (2015) that showed that exposing students to non-routine problems could lead to developing their critical thinking skills.

In terms of group 2 students, it is shown that there is a significant difference in the critical thinking skills of the students in terms of recognizing assumptions. It implies that the level of critical thinking skills of group 2 from authentic to non-linear strategy has a significant difference with a p-value of 0.000. It means that most students perform better when exposed to the authentic strategy of identifying the factual evidence presented and examining how relevant it is. It is also noted that the students feel it is easier to identify the mathematical concepts in the authentic strategy considering it has something to do with real life. Cai and Lester (2010) emphasized that in an authentic problem-solving environment, students may communicate their answers to their group or class in a manner that seems natural to them and learn mathematics via social interactions, meaning negotiation, and achieving a common understanding. As a result, students are given chances to clarify their thoughts and get new views on the topic or idea they are studying. However, there is no significant difference in the critical thinking skills of the students as to evaluating arguments and drawing conclusions. This means that regardless of the strategies being utilized in the study, they do not significantly affect students' performance in the critical thinking skills as to evaluate arguments and draw conclusions. Regardless of their exposure to the two strategies, the students' lowest critical thinking skill is found in evaluating arguments and drawing conclusions. Students' lack of critical thinking can be evident in their arguments where their explanations are insufficient, there are fewer logical assumptions, and there is less evaluation based on evidence (Muhlisin et al., 2016).

Table 4
Difference on the Critical Thinking Skills of the Students by Sequence

	N-A		A-N				
	M	**SD**	**M**	**SD**	**t**	**df**	**Sig. (2-tailed)**
Recognizing Assumption	2.31	0.77	2.08	0.44	1.013	28	0.320
Evaluate Arguments	2.12	0.68	1.97	0.56	0.685	28	0.499
Draw Conclusion	1.68	0.63	1.39	0.33	1.581	28	0.125

Table 4 displays the difference in the critical thinking skills of the students according to their sequence. The result shows that the mean in the first sequence (N-A) in terms of recognizing the assumption (2.31) is higher than the mean in the second sequence (A-N) (2.08). This indicates that the students in the first sequence perform better. However, these data were subjected to further statistical analysis, which revealed no significant difference (t=1.013; p=0.320) existed as to sequence.

In evaluating arguments, the mean in the first sequence (2.12) is higher than the mean in the second sequence (1.97). This demonstrates that students who belong in the first sequence performed better. Nevertheless, these data were statistically analyzed, and the results showed no significant difference (t=0.685; p=0.499) according to sequence.

In terms of drawing conclusions, the mean in the first sequence (1.68) is higher than in the second sequence (1.39). This reveals that the students in the first sequence perform better. Nonetheless, these data were statistically examined, and the findings indicated no significant difference in terms of sequence (t=1.581; p=0.125).

Overall, students' critical thinking skills have no significant difference regardless of which strategy they use first. This means that the order of strategy does not matter to the students' critical thinking skills. For that reason, it can be concluded that the sequence could not affect the level of the students' critical thinking skills and problem-solving. This is parallel to Bankole (2012), where they failed to make a definitive statement on teaching strategies that may help students develop critical thinking skills.

5. Conclusion

The main purpose of this study is to investigate pre-service teachers' critical thinking skills through problem-based learning strategies. This study reflects that evaluating arguments and drawing conclusions are low-skilled as to critical thinking skills, despite the fact that the respondents are mathematics pre-service teachers. It was found that there was a significant difference in Group 1 in the critical thinking skills in recognizing assumptions, evaluating arguments, and drawing conclusions. However, in Group 2, significant difference exists only as to recognizing assumptions. This study also revealed that there is no significant difference in the critical thinking skills of the students according to their sequence. This implies that students' critical thinking skills have no significant difference regardless of what strategy they use first.

Critical thinking skill among pre-service teachers is still lacking, particularly in terms of evaluating arguments and drawing conclusions due to a lack of prior information and a thorough comprehension of mathematics ideas. With this, the researcher recommends that future researchers build viable approaches for developing effective initiatives to promote critical thinking teaching and learning in mathematics education. They may also expose students, especially pre-service teachers, to different types of training in order to enhance their critical thinking skills.

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International Journal of Science, Technology, Engineering and Mathematics
Volume 2 Issue 2 June 2022
DOI: https://doi.org/10.53378/352890

English Language Proficiency and Geometric Proof Skills of Students

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Abstract

Educational reforms and curriculum development were continuously implemented for quality and inclusive education for all learners. As proficiency in English language became an issue, it has also been a burden in studying mathematics, specifically geometry, which is mainly written and taught in English Language. Hence, this study utilized descriptive correlational design in describing the students' proficiency in English language, specifically in reading comprehension, and Geometric proof skills in terms of correctness, appropriateness, logical reasoning, and clarity. Additionally, random sampling technique was used in choosing 30 mathematics students at a state university in Laguna. The quantitative data revealed that the respondents were proficient in terms of reading comprehension. However, with the presence of socio-cultural differences, their answers were influenced by environmental interference. On the other hand, the students performed advanced level in all aspects of Geometric proof skills. It was also found out that there is a significant relationship between reading comprehension and the correctness of proof and logical reasoning. Contrary to the results of the two components of geometric proof skills, reading comprehension has no significant relationship to both appropriateness and clarity. Based from the findings, the study suggests educators expose their students to a wide variety of reading materials in enhancing geometric proof skills.

Keywords: *English Language Proficiency, Geometric Proof Skills, Reading Comprehension, Mathematics*

Article History:

***Received**: March 4, 2022* ***Revised**: May 23, 2022*
***Accepted**: May 26, 2022* ***Published online**: June 1, 2022*

Suggested Citation:
Bombio, W. & Del Rosario, A.P. (2022). English Language Proficiency and Geometric Proof Skills of Students. *International Journal of Science, Technology, Engineering and Mathematics*, Volume 2 Issue 2, pp. 29 - 47. DOI: https://doi.org/10.53378/352890

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1. Introduction

In terms of reading and mathematics, the Philippines ranked last among 79 participating countries in the Program for International Student Assessment (PISA) 2018, with scores of 340 and 357, respectively, far exceeding the average of 487 for reading and 489 for mathematics (OECD, 2019). In order to achieve the goal of becoming a globally competent nation, education reforms were implemented with the goal of improving quality in the fields of science and mathematics. These subjects should be prioritized by administration and school systems in their program thrust (Akbasli et al., 2016).

Aside from its complexities and applications in scientific advancement, the beauty of mathematics involves an exclusively unique language unbounded by any cultural differences or socio-economical conflicts (Jourdain & Sharma, 2016). It employs a set of mathematical symbols and graphical representations to accomplish a specific purpose and function (Lin & Yang, 2007). However, problems and misunderstandings will arise when students experience contrast and information-clash when they bring up their prior knowledge when studying mathematics (Barwell, 2011). Aside from the difficulties encountered in the cognitive domain, mathematics presents significant difficulties in the use of language (Jourdain & Sharma, 2016). If a person does not understand English, he or she will not be able to understand a text written in English (Baful & Derequito, 2022). A person who knows nothing about English, for example, is always behind in analyzing information and technological discoveries that are mostly written in English Language (EL) (Racca & Lasaten, 2016). According to Aina et al. (2013), subjects such as mathematics and science necessitate extensive adaptation of language and its functions. The use of language is directly proportional to the effectiveness of one's thinking. As a result, language proficiency—in this case, EL—determines student performance in academic subjects, particularly mathematics.

One of the most important aspects of EL proficiency is reading comprehension (Racca & Lasaten, 2016). Some people read simply, but comprehension extends beyond the content of the text to include the secrets, stories, and context of a reading material (Akbasli et al. 2016). The reader and the text should communicate in a way that benefits both of them (Onkoba, 2014). Decoding symbols and solving problems using different arithmetic operations may not be enough in mathematics subject reading; students must first appreciate the holistic view of what they are reading before they understand how they are going to solve a problem or perform in this task, mathematically (Barwell, 2011). Language translation is another issue that arises when learning mathematics. Mathematics contains specialized symbols that make it difficult to interpret mathematical text. Learners becoming lost in translation is a result of issues with syntax, semantics, and language consistency while attempting to comprehend a specific math problem (Njagi, 2015). Because of the additional process of translating one text for it to be understandable, English language learners may fall behind native language speakers.

The study of Perez and Alieto (2018) focused on instilling the idea of preventing problems from occurring by constantly revising school systems and curriculum in the early grades. For instance, language shift from English to mother tongue is one of the Department of Education's curriculum amendments, in which simple mathematical concepts such as basic arithmetic operations are translated to the language in which the child is proficient. This has an impact on students in the early grades, specifically grades 1 to 3. According to Perez and Alieto (2018), addressing language issues in childhood and the early stages of learning becomes proactive in addressing potential problems in higher education and promotes progress in both mathematical achievement and proficiency. According to Onkoba (2014), lack of mastery in various fields of mathematics does not directly determine a student's performance in the subject; rather, poor performance is the result of poor acquisition of language skills. Similarly, poor language skills predictably lead to poor mathematical skills. This was congruent to several studies that English Language Proficiency (ELP) predicts academic excellence in mathematics (Henry et al., 2014; Racca & Lasaten, 2016).

Some curricula for education students majoring in mathematics were designed to begin with the fundamentals—functions and definitions of mathematical theorems, properties, or propositions—in their first and second years of study. Meanwhile, junior and senior years require a higher level of reasoning, proof-skills, and the ability to apply knowledge gained in the first two years of learning. Learning mathematics will be difficult if EL is difficult to use and ELP is lacking (Rambely et al., 2013). This shortcoming may be taken into account and result in danger, as well as another shortcoming, a few years later while they are teaching. Although the PISA 2018 results placed an emphasis on mathematical literacy and reading comprehension, they do not directly include mathematical proof skills, particularly geometric proofs skills (Sälzer & Roczen, 2018). However, Gunhan (2014) found that reading comprehension, along with reasoning and proof-making skills, determines mathematical achievement. Geometric problems are designed to assess cognitive skills in problem solving or reasoning, resulting in an assessment of reading ability in mathematics. Individuals who argue in a reasonable manner are also those who produce accurate proofs and communicate them to others in written form (Gunhan, 2014; Baful & Derequito, 2022).

Some notable recommendations from previous research studies include establishing relationships between English language proficiency and a much more specific field of mathematics. Gunhan (2014) emphasizes the effects of language proficiency in reasoning in his study and then connects it to how students reason out in mathematics by making proofs. Higher thinking requires both language skills and mathematical reasoning, such as proving geometric figures and other geometry problems (Perez & Alieto, 2018). To fill the gaps previously identified in this study, this study sought to understand the relationship between pre-service mathematics students' English language proficiency and geometric proof skills. This group of students was chosen specifically because they are actively immersed in the areas of Geometry and English as part of their preparation to be future mathematics teachers.

2. Literature review

2.1. Cognitive Academic Language Proficiency

The study was founded on the concept of Cognitive Academic Language Proficiency (CALP), which was coined in 1979 by Cummins (Macaro et al., 2018) and later became the central idea of Krashen and Rosenthal (Long, 2014) studies that determined the relationship between students' cognitive and linguistic processes and their academic performance. Language can be difficult for English language learners because it does not eliminate the impediment of cultural differences. Although the mathematics subject was unbounded by these differences due to universally accepted functions and symbols, proving geometry problems was not limited to specific functions; it also discussed logical reasoning and other aspects (Barwell, 2011).

CALP is a formal approach to introducing language proficiency in an academic setting. It is required in classrooms and educational institutions for activities such as reading, writing, joining and participating in formal conversations, and even taking quizzes and exams. As a result, students who do not incorporate CALP into their development are more likely to struggle with subjects such as mathematics and other academic subjects (Long, 2014). While learning their first language or mother tongue, all children developed BICS. Meanwhile, CALP is acquired through a series of cognitive processes in which the individual engages during his or her learning journey (Racca & Lasaten, 2016). This study concentrated on CALP.

The findings of Krashen and Rosenthal (Macaro et al., 2018) demonstrate that problems with CALP and language proficiency can lead to deficiencies in mathematics learning. Reading comprehension was one of the dimensions of language proficiency (Zheng & Cheng, 2008). On the other hand, the method of assessing students' geometric proof skills was dynamic and relative in nature. Although interpretations may change depending on the students' progress and proficiency, the following dimensions will remain in proof making: correctness, appropriateness, logical reasoning, and clarity (Balacheff, 2008). As a result, the CALP theory gave the researcher a reason to investigate the relationship between students' English language proficiency and geometric proof skills.

2.2.Reading Comprehension

Reading comprehension extends beyond the content of the text to include the secrets, stories, and context of the reading material (Akbasli et al., 2016). Reading comprehension in mathematics involves not only decoding symbols and solving problems using various arithmetic operations, but also appreciating the holistic view of what they are reading before they understand how they are going to solve a problem or perform in this task, mathematically (Barwell, 2011).

Mathematics is known for its specialized symbols, which make it difficult to interpret mathematical text. Learners are getting lost in translation because of issues with syntax, semantics, and language consistency while attempting to understand a specific math problem (Njagi, 2015). Because of the additional process of translating text for understandability, English language learners are one step behind their native language speakers' counterparts. In terms of mathematical translation, graphical and symbolic representations are converted to mathematical equations or functions, and vice versa (Malanog & Aliazas, 2021). Unlike language translation, which has a bias in its native speaker, mathematical translation is unbounded by culture and language and can only be determined by the student's skills and systematic factors such as errors in representation or an incomplete understanding of the text (Adu-Gyamfi et al., 2012).

People regard writing in academe as grammatically complex and structurally listed in detail, such as research writing, proof writing, or even working on problems. However, before educating, the key to writing is communication; people cannot comprehend a complex structure if they do not first understand it (Biber et al., 2016). Writing can be examined through a composition based on the information given through sentences outline, pictures, and so on, thanks to technological advancement and the presence of infographics, graphs, and visual aids (Zheng & Cheng, 2008). Furthermore, writing is a complex tool that can change depending on the social context. Spence (2010) describes a learner who grew up in an English-speaking environment with far more potential than a learner who does not. Language combinations in terms of listening, speaking, reading, and writing are rooted in social situations, historical events, and sometimes norms.

Reading assesses students' abilities to comprehend written texts in nature. Reading comprehension can be measured using two components (Zheng & Cheng, 2008). The first is in-depth reading, which is defined as the breadth of one's word knowledge. It indicates how much a person understands rather than how much a person knows about something. Reading in depth implies a significant difference from summary writing (Li & Kirby, 2015). Skimming and scanning, according to Sari (2016), are useful methods for reading quickly and effectively. Skimming is reading the entire reading material in order to determine its nature and how it is organized, whereas scanning is quickly reading through the text in order to find a specific piece of information.

2.3.Correctness

Correctness necessitates completeness. The key to determining the correctness of a proof in writing mathematical proof—specifically in geometric proofs—is its completeness. Proofs are a series of reasons in chronological order that cannot be simply jumbled out (Lee, 2012). In some cases, the premise "congruent parts of congruent triangles are congruent" cannot be used to prove the congruency of two triangles without first proving that the triangles are congruent. The polished product—the end result—is the correct proof we see. The trials and errors of proving a specific statement or premise cannot be included in the definition of proving. Rather than being creative, we define correct as something that is only right and proper (Selden & Selden, 2015).

2.4.Appropriateness

The process of analyzing appropriateness includes arguments such as answering questions that examine the property of the proof methods, affirming and denying claims based on truth value, using acceptable theorems and polygon definitions, and a conscious assessment of whether a proof or something was correct or not (Selden & Selden, 2015). Appropriateness and correctness were diametrically opposed.

Appropriate does not imply correct. For example, the statement to be proven could be that two triangles in a quadrilateral are congruent. The statement can be proven using postulates and congruency properties. However, there may be properties of a quadrilateral that are misleading or inappropriate and can be removed during the proof process.

2.5.Logical Reasoning

According to Gunhan (2014), logical reasoning is the process of reasoning and justifying arguments in order to solve or contribute to the solution of a mathematical problem. Logical reasoning was gained by the situation in which they are forming their own conjunctures, which is rooted in elementary school level. Secondary level students improved on this by reasoning inductively and deductively after evaluating their own conjunctures. As a result of their lack of reasoning skills, learners tend to see mathematics as a set of rules that must be strictly followed rather than reasoned out for more flexible and complex learning. It is always critical that a claim be supported and defended by carefully constructed arguments. A good reasoning skill, on the other hand, should be relevant and have a clear manifestation in the classroom environment all the way up to the communities. Logical reasoning in geometry should extend beyond what is drawn, presented in figures, or written in books (Adu-Gyamfi et al., 2019).

2.6. Clarity

The fact that a proof is clear and understandable to the intended audience is proof of good proof-making and mathematical writing. Because there are numerous and diverse mathematical books that can be used as references, clarity in notation may differ in proving. However, there is a neutral common point where general clarity is required. For example, in the case of proofs other than direct proof, the writer who will create the proofs should specify the type of proof he or she will use. Abbreviations should be avoided when writing proof because they impede understanding between the reader, particularly if the reader is inexperienced (Lee, 2012).

Precision is essential when writing proofs. To avoid misinterpretation and confusion, each mathematical statement should have a distinct meaning. Every proof should have a beginning and an end. Learners should be able to tell where your proof begins and ends (Lee, 2012). The ability to communicate in English determines mathematical achievement at the elementary and secondary levels of teaching and learning (Barley, 2011; Racca & Lasaten, 2016; Henry et al., 2014; Stoffelsma & Spooren, 2017; Rambely et al., 2013). A high level of mathematical achievement in childhood predicts a high level of mathematical reasoning skill later in life. Proof-making requires mathematical reasoning (Gunhan, 2014). However, no studies have been conducted to demonstrate that these three variables are transitive. A good command of the English language does not always imply a strong grasp of mathematics. However, based on the related literature, there should be a collaborative effort between the institution, as they control the content of education, teachers as they employ new and more efficient teaching methods, and students as they determine their pathways of deeper and purposeful learning.

3. Methodology

The purpose of this study was to look at the relationship between students' English language skills and their geometric proof skills. Its specific goal is to assess students' English Language Proficiency in reading comprehension as well as their geometric proof skills in terms of correctness, appropriateness, logical reasoning, and clarity. Finally, it attempted to establish a link between students' English language proficiency and geometric proof skills.

The study used the descriptive-correlational research design to answer questions in the problem statement and purpose of the study. The descriptive research design aims to observe a specific phenomenon rather than to answer whys and other questions about what is going on (Gravetter & Forzano, 2019). Correlational research design, on the other hand, seeks to discover systematic relationships between variables. It assesses two or more variables relevant to and related to the study (Gravetter & Forzano, 2019).

The study's respondents were second and third-year Bachelor of Secondary Education students majoring in Mathematics. These students were chosen specifically because they are currently preparing and training to teach mathematics in the Basic Education program. Using a simple random sampling technique, the study identified thirty (30) students from both the second and third years of education who specialized in mathematics.

The modified tests that revolve on measuring the English language proficiency and geometric proof skills of mathematics students were the main instruments used in gathering data for this study. They were assessed using a battery of English language proficiency tests. The testing method was adopted and modified to meet the needs of the study (Zheng & Cheng, 2008). The study restricted the English language proficiency variable to reading comprehension only. Furthermore, the study adopted and modified the Copes-validated and used rubric for determining skills in geometric-proofs (Priest et al., 2013). To determine the level of English Language Proficiency, the series of reading comprehension tests about English language proficiency, consisting of three (3) tests in total and thirteen (13) questions each, was created. On the other hand, in order to determine the level of Geometric Proof Skills, the test that will measure the geometric proof skills was also created. Three (3) geometry proving problems on isosceles triangles, rhombuses, and parallel lines comprise the test.

The study sought the expertise of three English teachers for the English Language Proficiency Test and three Mathematics teachers for the Geometry Problems Test to ensure the content validity of both the adopted and modified tests. Validators for the English language proficiency research instrument corrected all grammatical errors in both the article and problem sets. They also eliminated all of the questions that they believed were more difficult than the others, reducing the number of questions on each test from fifteen (15) to thirteen (13). Meanwhile, validators with geometric proof skills suggested that the number of problem sets be increased. The number of problem sets, specifically geometry problems, was increased from two (2) to three (3). Another validation procedure was then applied to the additional problem.

Following the validation of the research instrument, the letter of approval was sent and permission was requested to administer the research to the college dean as well as the respondents. Following approval from the college dean, the English Language Proficiency tests were administered to all study participants. Following the completion of the three (3) parts of 13-question problems, the geometry proof problems and the three (3) given problems by two-column proofs were also administered. Google Forms was used to administer all of the tests. The test questionnaires were collected thereafter.

The current study employed both descriptive and inferential statistics. Frequency and percentage were used in response to the descriptive analysis on English Language Proficiency and Geometric Proof Skills. Spearman rank correlation was used to answer the inferential question of whether there is a significant relationship between students' English Language Proficiency and Geometric Proof Skills.

The study ensured confidentiality of the results and the respondents' personal information. It was also optional for respondents to provide their names.

4. Findings and Discussion

Table 1

Students' English Language Proficiency Level in terms of Reading Comprehension

Scores	Frequency	Percent	Interpretation
	Test 1		
11-13	14	46.6	**Exemplary**
8-10	13	43.3	**Proficient**
5-7	1	3.3	**Developing**
0-4	2	6.6	**Emerging**
	Test 2		
11-13	4	13.3	**Exemplary**
8-10	10	33.3	**Proficient**
5-7	10	33.3	**Developing**
0-4	6	20.0	**Emerging**
	Test 3		
11-13	4	13.3	**Exemplary**
8-10	12	40.0	**Proficient**
5-7	12	40.0	**Developing**
0-4	2	6.6	**Emerging**

Table 1 presents the respondents' English language proficiency in terms of reading comprehension for all the three tests.

In the first test, it can be seen that fourteen (14) respondents (46.6 %) out of thirty (30) achieve an exemplary level. This means that the majority of respondents have an exemplary level of reading comprehension. Students who perform exceptionally well receive scores ranging from 11 to 13. This demonstrates that the majority of students understood the reading materials and the corresponding questions with little to no error. On the other hand, 13 students or 43.3 percent are proficient, one (1) or 3.3 percent is developing, and two (2) students or 6.6 percent of respondents received 0-4 scores in the emerging level. Average scores can also be used to describe the proficient level. Students at this level answered questions with consistency and accuracy, but they missed some difficult questions. In comparison to the rest of the respondents, the two students at the emerging level did not exhibit clear patterns in their responses.

The instrument in test 1 is an article about scientific facts and cannabis plant terminologies. Similar to mathematics, cultural differences have no bearing on scientific facts and terminologies (Jourdain & Sharma, 2016). There will be no internal conflict or other factors that could influence how respondents answer the test.

In test 2, it is revealed that 10 (33.3 %) of respondents are proficient and developing, emerging, 6 (20 percent) of respondents, and only 4 (13.3 %) of respondents are exemplary. The difference in level of scores and frequencies between tests 1 and 2 is very noticeable. The frequency of the student who performs exemplary drops from fourteen (14) to four (4), in terms of percentage, the percentage decreases by one-third (33.3%), while the students who are in developing level increased by nine (9) from one (1) accounting to nearly 30 percent of de-escalations.

The research of Akbasli et al. (2016) suggests possible explanations for sudden changes in score frequencies. They stated that there are times when testing reading comprehension that the students' prior experience and current knowledge of a topic influence how well they comprehend a written article. For example, the research instrument for test 1 is about scientific facts and terminologies about cannabis and the marijuana plant, whereas the research instrument for test 2 is about China's territorial usurpation in countries such as Myanmar, Malaysia, and Japan. Personal biases of respondents may hinder and overlap the idea presented in the article because context differences are present and much more observable in test 2.

In test 3, the proficient and developing levels have equal frequencies, accounting for (12) twelve, or 40% of the respondents in each category. Furthermore, (4) four students, or 13.3 %, reach the exemplary level, while (2) two students, or 6.6 %, fall into the emerging level. Students at the proficient level are not expected to perform exceptionally well in the presence of an article that is completely unfamiliar to the respondent's profile; rather, they are expected to perform averagely. The results of the respondents' scores mirrored the researcher's expectations. When compared to test 1, the results of test 3 are much more similar to those of test 2. In comparison to the results of test 2, the frequencies for developing and proficient level both increase by 6.6 percent

(13.3 % in total), with the four (4) students from emerging level accounting for this increase. There are some shared similarities between the articles on test 2 and test 3. The second test focused on China's territorial usurpation of Southeast Asian countries, while the third examines the methods of evaluating a manager in a company. Both are uncommon in the respondent's daily life. China's move is distinct from that of other countries, and methods of assessment in the workplace differ in terms of teaching. According to Spence (2010), language proficiency in terms of reading comprehension is a result of social differences. Reading proficiency can be influenced by social situations, historical events, and sometimes norms.

Table 2

Students' English Language Proficiency Level in terms of Reading Comprehension in Series of Tests

Scores	Frequency	Percent	Interpretation
11-13	5	16.6	**Exemplary**
8-10	16	53.3	**Proficient**
5-7	6	20.0	**Developing**
0-4	3	10.0	**Emerging**
Total	30	100.0	

Table 2 summarizes the frequencies of the average (mean) scores of students in the series of reading comprehension tests. More than half of the respondents, or 16 students (53.3 percent), demonstrated a proficient level of performance. In a series of tests, the majority of the students' average scores fall in the proficient range. When compared to the results of the second and third tests, the percentage of students performing at the proficient level increased. This is due to their performance on the first test, in which the majority of students performed admirably. When the average scores were computed, some of the developing level scores were elevated one category above their usual scores.

Meanwhile, six students (20% of the respondents) perform at the developing level, five (5) or 16.6% perform at the exemplary level, and three (3) represent 10% perform at the emerging level. Even if students are proficient in reading comprehension, problems with consistency or retention may arise when students encounter contrast and information-clash when applying their prior knowledge to the article they are reading (Barwell, 2011). In terms of testing, the difficulty or nature of the tests has a significant impact on students' reading comprehension (Baful &

Derequito, 2022). Language teachers should expose their students to a wide range of reading materials in order to improve reading comprehension in terms of level and consistency.

Table 3

Students' Level of Geometric Proof Skills in terms of Correctness

Scores	Frequency	Percent	Interpretation
	Correctness		
3	17	56.7	**Advanced**
2	12	40.0	**Proficient**
1	1	3.3	**Beginner**
	Appropriateness		
3	19	63.3	**Advanced**
2	10	33.3	**Proficient**
1	1	3.3	**Beginner**
	Logical Reasoning		
3	19	63.3	**Advanced**
2	10	33.3	**Proficient**
1	1	3.3	**Beginner**
	Clarity of Notation		
3	18	60.0	**Advanced**
2	9	30.0	**Proficient**
1	3	10.0	**Beginner**

Table 3 shows the students' geometric proof skills in all the four skills tested.

In terms of correctness, more than half of the respondents, 17 out of 30 (56.7 %), have advanced accuracy in their geometric proof skills. Students who advanced in their studies were able to provide correct and complete proof. Although the percentage of students who perform at the advanced level of geometric proof skills in terms of correctness is 56.7 %, which is a very large portion of the respondents, the frequency of students who perform at the advanced level is the lowest when compared to the other three components of geometric proof skills. Furthermore, 12 students (40 % of the respondents) have proficient skill levels, while only 1 student (3.3 %) falls short of the beginner level of correctness. Almost all of the respondents who reach the proficient level struggle to complete their proofs. Even if the respondents' proof is on the right track, they would have difficulty closing out their proofs. Students with proficient skill levels can solve or prove the problem; however, they can be misguided while answering, resulting in an incomplete proof or even a significant unjustified leap in the answers. The only student at the beginner level was unable to submit an answer in all three problem sets, and it is possible that the student was unable to answer even one of them, given that the respondent's choice is pre-service math teachers.

According to Lee (2012), completeness is required in determining whether or not the proof is correct. Some proofs include irrelevant information and statements, while others lack relevant information. To increase the number of students performing at the advanced level, the teacher should place a greater emphasis on teaching complete and correct proofs.

In terms of appropriateness, 19 students, or nearly two-thirds of the class (63.3 %), demonstrate advanced level of skill in terms of selecting appropriate methods in their ways of proving. Even if the respondents demonstrate an advanced level of appropriateness, they share methodological similarities in their responses. There are no indications of a different approach. In terms of appropriateness, the 63.3 % frequency of respondents who reach advanced level of geometric proof skills can have a variety of implications. First, the students excel at selecting the most appropriate—that is, the simplest and shortest—methods and statements for proving their points (Selden & Selden, 2015). Another implication is that students are only taught the most appropriate methods of proving. Even though geometric proofs are non-routine problems that require creativity and critical thinking, they can sometimes be answered routinely (Dio, 2021). Problems with the latter implication may arise if students pursue a higher level of mathematics (Andrade & Pasia, 2020). Furthermore, 10 students (33.3 %) perform at the proficient level, while only 1 student (3.3 %) performs at the beginner level in terms of appropriateness. 10 students perform competently in terms of appropriateness; these students were able to select appropriate proof methods, but did not master them, resulting in incorrect assumptions in proofs. Similarly, only one (1) student performed at the beginner level in all components and was unable to answer the problem sets.

In terms of logical reasoning, 19 (63.3 %) of the respondents have advanced logical reasoning skills. Students with advanced logical reasoning abilities performed clear and correct statements in chronological order. The proper statement-reason arrangement accounts for the high frequency of students with high levels of logical reasoning (Mulligan, 2015). Enhanced logical reasoning has its roots in elementary and secondary school, where students improve their reasoning skills by forming conjectures and reasoning inductively or deductively based on those conjectures (Gunhan, 2014). For example, first and second year college students' curricula focused on improving their fundamental math skills. The concept of line and polygon properties was discussed and taught as early as secondary school. In higher levels of mathematics, underdeveloped reasoning skills in childhood show rigid and narrow reasoning skills (Lowrie, Logan, & Ramful, 2016). Furthermore, 10 students (33.3 %) reason logically at the proficient level. The effect of having a proficient level in terms of logical reasoning is focused on the readers' ability to read the proof itself, rather than on the individual's skills. In the eyes of the evaluators, who are mathematics experts by profession, the proofs of students who perform proficiently were logically correct. When presented to someone who is not mathematically inclined, the proof is incomprehensible. These aspects or factors are critical in pre-service mathematics teachers because they will soon be in the classroom.

Furthermore, only one student (3.33%) performs at the beginner level of logical reasoning. The student did not complete proofs in all of the problem sets. Teachers and educational institutions should focus more on developing primary and secondary students' reasoning skills in order to retain and improve their students' logical reasoning.

In terms of clarity of notation, the highest frequency is still on the advanced level, which corresponds to 18 students and 60 %. The shift in educational reforms resulted in a high number of frequencies for advanced level geometric proof skills in terms of clarity (Rambely, Ahmad, Majid, & Jaaman, 2013).

In terms of clarity, nine (9) students, or 30% of the respondents, perform at the proficient level, while three students, or 10% of the respondents, perform at the beginner level. Clarity was assessed based on how respondents practiced accuracy in notation, such as assigning names to lines and angles, as well as the connection of the statement to the reason. The proof of the students with proficient levels is understandable in a simple manner, with the exception of a few parts that are unclear. Most students at the proficient level have a tendency to state and justify reasons incorrectly based on their corresponding statements. Furthermore, the proofs of students who performed at the beginner level are perplexing and disorganized. Beginners struggle to connect statements to their reasons and to use mathematical symbols.

The emphasis on discussing the fundamentals among first and second year students, such as functions and definitions of mathematical theorems, properties, or propositions, becomes apparent in measuring achievement in higher math subjects such as proving. This implies that educational institutions should continue to emphasize the fundamentals and basics of basic mathematics in the early years of study (Rambely et al., 2013).

Table 4

Test of Relationship between English Language Proficiency and Geometric Proof Skills Components

		Geometry Proof Skills			
		Correctness	Appropriateness	Logical reasoning	Clarity
Spearman's rho	**English language Proficiency**	0.469**	0.327	0.434*	0.354

**. Correlation is significant at the 0.01 level (2-tailed).
*. Correlation is significant at the 0.05 level (2-tailed).

Table 4 shows the relationship between students' English Language Proficiency and Geometric Proof Skills. Accordingly, there is a significant positive relationship between English language proficiency and geometric proof skills. It demonstrates a relationship between English language proficiency and the two components of geometric proof skills, namely correctness and logical reasoning. The findings support and align with Gunhan (2014) that geometry problems can influence students' reading comprehension and reasoning skills. Furthermore, the study connects issues of English language proficiency in terms of reading comprehension to reasoning and proof-making skills.

The correctness of the proof is related to the students' reading comprehension. Meanwhile, there are no clear links that correctness as a geometric proof has a relationship with English language proficiency. Geometry problems can benefit from reading comprehension. Proof-making, as defined by correctness, is the process of arranging things in the correct and proper order (Priest et. al., 2013). Teachers can improve geometric proof skills in the same way that they can improve reading comprehension by constantly teaching a wide variety of geometry proof problems. In terms of appropriateness and clarity, there is no significant relationship between English language proficiency and geometric proof skills.

The following are some of the possible reasons for the components such as appropriateness and clarity in the results. Respondents exhibit personal biases in answering reading comprehension articles that address socio-cultural text that is completely different from them, as the findings for reading comprehension imply (Barwell, 2011). Because of the issues in bringing their own personality in answering, this makes sense to the relationship of reading comprehension to correctness and logical reasoning. For issues of appropriateness and clarity, the way these components are scored and measured is based on students' use of mathematical symbols and concepts in mathematics—these sets of symbols only serve specific functions and purposes and are unbounded by socio-cultural differences (Lee, 2012; Lin & Yang, 2007). Future researchers should look into other aspects of English language proficiency besides reading comprehension. Furthermore, future researchers should attempt to correlate reading comprehension to proofs in various branches of mathematics, such as logic or number theory.

5. Conclusion

The primary goal of this research is to look into the relationship between English language proficiency and geometric proof skills. Reading comprehension was discovered to have a significant relationship to both dimensions of geometric proof skills, such as correctness and logical reasoning. Reading comprehension, on the other hand, has no significant relationship with both appropriateness and clarity. Adjustments and actions are suggested in light of the study's findings and conclusions. To improve students' reading comprehension skills and consistency, the study suggests incorporating a wide range of different reading materials into teaching instructions. Similarly, further study could increase the number of respondents to strengthen the accuracy and validity of the outcome. Further studies can also use various proving strategies or proving in other branches of mathematics such as logic and number theory.

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